Tôhoku Math. J.
45 (1993), 437–446

WEIGHTS FOR THE ERGODIC MAXIMAL OPERATOR AND A. E. CONVERGENCE OF THE ERGODIC AVERAGES FOR FUNCTIONS IN LORENTZ SPACES

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(Received March 17, 1992, revised September 2, 1992)

Abstract. In this paper, we deal with an invertible null-preserving transformation into itself of a finite measure space. We prove that the uniform boundedness of the ergodic averages in a reflexive Lorentz space implies a.e. convergence. In order to do this, we study the "good weights" for the maximal operator associated to an invertible measure preserving transformation.

1. Introduction and results. Let T be an invertible measure preserving transformation on a measure space $(X, \mathscr{M}, \mu)$. Let $T_{n,m}$ and M be the ergodic averages and the maximal operator defined, respectively, by

$$T_{n,m}f(x)=\frac{1}{n+m+1}\sum_{j=-n}^{m} f(T^j x) \quad \text{and} \quad Mf=\sup_{n,m\geq 0} T_{n,m}|f| .$$

Martín-Reyes [6] studied the good weights for M to be bounded in L_p $(1<p<\infty)$ and from L_p to $L_{p,\infty}$ $(1\leq p<\infty)$. He proved that M is bounded from $L_p(v)$ to $L_{p,\infty}(u)$ if and only if (u, v) satisfies $A_p(T)$, which means for $p>1$

$$\left(\sum_{i=0}^{k} u(T^i x)\right)\left(\sum_{i=0}^{k} v^{1-p'}(T^i x)\right)^{p-1}\leq C(k+1)^p \quad \text{a.e.}$$

with C independent of k and x and $pp'=p+p'$, and for $p=1$

$$Mu(x)\leq Cv(x) \quad \text{a.e.}$$

Moreover, he proved that, for $u=v$ and $p>1$, $A_p(T)$ is also equivalent to the boundedness of M in $L_p(u)$. Then, he used these results to obtain theorems about convergence a. e. of the ergodic averages of functions in weighted L_p-spaces.

Gallardo [2], [3] has generalized these results to Orlicz spaces.

Our purpose is to extend the L_p results to $L_{p,q}$ spaces. In this paper, we characterize

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1991 *Mathematics Subject Classification*. Primary 28D05.

Key words and phrases: Ergodic averages, ergodic maximal operator, Lorentz spaces, measure preserving transformations, null-preserving transformations, weights.

This research has been partially supported by D.G.I.C.Y.T. Grant PB88-0324.