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HOMOLOGY COVERINGS OF RIEMANN SURFACES

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Abstract. We extend a result on homology coverings of closed Riemann surfaces due to Maskit [1] to the class of analytically finite ones. We show that if S is an analytically finite hyperbolic Riemann surface, then its conformal structure is determined by the conformal structure of its homology cover. The homology cover of a Riemann surface S is the highest regular covering of S with an Abelian group of covering transformations. In fact, we show that the commutator subgroup of any torsion-free, finitely generated Fuchsian group of the first kind determines it uniquely.

1. The main theorem. Let S be a Riemann surface. We say that S is *analytically finite* if S is conformally equivalent to the complement of a finite number of points on a closed Riemann surface $\bar{S}$. If the genus of $\bar{S}$ is g and the number of deleted points is k, then we say that S has signature $(g, k; \infty, \ldots, \infty)$.

We say that an analytically finite Riemann surface S of signature $(q, k; \infty, \ldots, \infty)$ is *hyperbolic* if its universal covering surface is the hyperbolic disc. It is the case if and only if $2g-2+k>0$.

A Riemann surface $\hat{S}$ is an *Abelian cover* of S if there exists a regular covering $\pi: \hat{S} \to S$ with an Abelian group of deck transformations. The *homology covering* of S, $\pi: \tilde{S} \to S$, is the highest Abelian covering of S, that is, it is the covering corresponding to the commutator subgroup of the fundamental group $\Pi_1(S)$ of S.

THEOREM. *Let S_1 and S_2 be analytically finite hyperbolic Riemann surfaces of signature $(g_1, k_1; \infty, \ldots, \infty)$ and $(g_2, k_2; \infty, \ldots, \infty)$, respectively. Suppose S_1 and S_2 have conformally equivalent homology covering surfaces. Then S_1 and S_2 are conformally equivalent.*

REMARKS. The above theorem, for the class of closed hyperbolic Riemann surfaces, was proved by Maskit in [1]. Since the homology cover of a Riemann surface S, obtained by deleting one point in a closed Riemann surface $\bar{S}$, is the homology cover of $\bar{S}$ minus the orbit of a (suitable) point, the above theorem for $k_1 = k_2 = 1$ is an easy consequence of Maskit's result.

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