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THE TRACE THEOREM ON ANISOTROPIC SOBOLEV SPACES

Dedicated to Professor Takesi Kotake on his sixtieth birthday

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Abstract. The trace theorem on anisotropic Sobolev spaces is proved. These function spaces which can be regarded as weighted Sobolev spaces are particularly important when we discuss the regularity of solutions of the characteristic initial boundary value problem for linear symmetric hyperbolic systems.

In this note, we give the trace theorem on anisotropic Sobolev spaces which appear in the study of the initial boundary value problem for linear symmetric hyperbolic systems with characteristic boundary. The function spaces with which we concern ourselves will be denoted by $H_*^m(\Omega)$, Ω being an open set in $\boldsymbol{R}^n$ lying on one side of its boundary. Denoting the usual Sobolev space by $H^m(\Omega)$, we have the continuous embedding $H^m(\Omega) \hookrightarrow H_*^m(\Omega)$ for $m=0, 1, \dots$. $H_*^m(\Omega)$ is anisotropic in the sense that the tangential derivatives and the normal derivatives are treated in different ways in this space. In contrast with the case where the boundary is non-characteristic, the solution of the characteristic initial boundary value problem for symmetric hyperbolic systems lies in general in $H_*^m(\Omega)$, not in $H^m(\Omega)$. The trace theorem on $H_*^m(\Omega)$ is needed especially when we consider the compatibility condition. This is the motivation for the present work.

For simplicity, we suppose that Ω is a half-space in $\boldsymbol{R}^n$. Let

$$\boldsymbol{R}_+^n = \{(t, y) \mid t>0, y \in \boldsymbol{R}^{n-1}\} .$$

Let $\rho \in C^\infty(\overline{\boldsymbol{R}_+})$ be a monotone increasing function such that $\rho(t)=t$ in a neighborhood of the origin and $\rho(t)=1$ for any t large enough. By means of this function, we define the differential operator in the tangential directions

$$\partial_{\tan}^{(r,\alpha)} = (\rho(t)\partial_t)^r \partial_1^{\alpha_1} \cdots \partial_{n-1}^{\alpha_{n-1}} ,$$

where $\alpha=(\alpha_1, \dots, \alpha_{n-1})$, $\partial_i = \partial/\partial y_i$, $1 \le i \le n-1$. The differential operator in the normal direction is ∂_t^k. We fix a nonnegative integer m. Let $u \in L^2(\boldsymbol{R}_+^n)$ satisfy

$$\|u\|_{m,*}^2 = \sum_{r+|\alpha|+2k \le m} \|\partial_{\tan}^{(r,\alpha)} \partial_t^k u\|^2 < \infty ,$$

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