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THE SET OF SOLUTIONS FOR CERTAIN SEMILINEAR HEAT EQUATIONS

Dedicated to Professor Takeshi Kotake on his sixtieth birthday

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Abstract. We show that bounded solutions of an initial(-boundary) value problem for certain semilinear heat equations always come from the corresponding ordinary differential equations. As a consequence we immediately get a theorem of Kneser's type which was showed by Ballotti and Kikuchi in different methods.

1. Introduction. Kneser's theorem [5] is a famous result on ordinary differential equations concerning the structure of the set of all solutions for an initial value problem. Recently Ballotti [1] and Kikuchi [4] established a theorem of Kneser's type for the initial(-boundary) value problem of a semilinear heat equation

$$\begin{cases} u_t=\Delta u+\sqrt{u} & \text{in} \quad \Omega\times(0, T)\,, \\ u|_{t=0}=0 & \text{in} \quad \Omega\times\{0\}\,, \\ \dfrac{\partial u}{\partial\nu}=0 & \text{on} \quad \partial\Omega\times(0, T) \quad \text{if } \partial\Omega\neq\varnothing\,. \end{cases}$$

Here Ω is a bounded domain with smooth boundary, or $\boldsymbol{R}^n$. When $\Omega\neq\boldsymbol{R}^n$, ν denotes a unit outer normal vector of $\partial\Omega$. Let $L^p(\Omega)$ be the usual Lebesgue space, and $BC(\bar{\Omega})$ the set of bounded continuous functions on $\bar{\Omega}$. Their result is as follows:

THEOREM 1.1 (Kneser's theorem, cf. [1], [4]). *Let $X=L^p(\Omega)$ $(1<p<\infty)$ when $\Omega\neq\boldsymbol{R}^n$, and $X=BC(\boldsymbol{R}^n)$ when $\Omega=\boldsymbol{R}^n$. Then the set of (mild) solutions in $C([0, T]; X)$ is compact and connected in the class. Hence the cross-section of (mild) solutions in $C([0, T]; X)$ is compact and connected in X.*

Their proofs are based on arguments on evolution equations or partial differential equations. The theorem for partial differential equations, however, easily follows from the corresponding theorem for ordinary differential equations, if we can prove that all solutions belonging to $C([0, T]; X)$ are independent of the space variables. In this article we show that it is in fact the case for $X=BC(\bar{\Omega})$, which is the same setting as in [4] when $\Omega=\boldsymbol{R}^n$. Our method is applicable to the following problem:

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