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AUTOMORPHISMS OF SIMPLE CHEVALLEY GROUPS OVER Q-ALGEBRAS

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Abstract. We show that a simple adjoint Chevalley group over a Q-algebra without zero divisors and its elementary subgroup have isomorphic automorphism groups which are generated by the inner automorphisms, the graph automorphisms and the ring automorphisms. This leads to an expression for every automorphism as the composite of a ring automorphism and an automorphism of an algebraic group, which is analogous to the Borel-Tits theorem and the Margulis theorem for the automorphisms of rational subgroups of algebraic groups over certain fields.

Introduction. Let G be a simple Chevalley-Demazure group scheme of adjoint type. The main purpose of this paper is to describe the automorphisms of the Chevalley group $G(R)$ and its elementary subgroup $E(R)$ provided that the rank of G is greater than one, where R is an associative and commutative algebra over the rational number field Q without zero divisors. The first study of this problem goes back to the work of Schreier and Van der Waerden [13] where they gave a description of the automorphisms of the projective group PSL_n over an algebraically closed field. The automorphisms of adjoint simple Chevalley groups were determined first by Steinberg [14] for finite fields and then by Humphreys [12] for infinite fields. We refer to [11] for a historical survey on homomorphisms of algebraic groups and Chevalley groups. In this paper we discuss the automorphisms of all subgroups between $G(R)$ and $E(R)$. It turns out that such an automorphism can be always expressed in a unique way as a product of an automorphism induced by the conjugation of an element in $G(R)$, a graph automorphism and a ring automorphism (see §1 for the definition). We find that each automorphism of a subgroup between $G(R)$ and $E(R)$ is a restriction of an automorphism of $G(R)$ and, meanwhile, keeps $E(R)$ invariant (see Theorem 1). This leads to an isomorphism between the automorphism group of $G(R)$ and the automorphism group of $E(R)$. The structure of the automorphism group of $G(R)$ and $E(R)$ is given by Theorem 2. We show in Theorem 3 that every automorphism of a subgroup between $G(R)$ and $E(R)$ is a composite of a ring automorphism and an automorphism as an algebraic group, which is an analogue of the Borel-Tits theorem [4] and the Margulis theorem [17]. When the rank of G is equal to one, the automorphisms of $G(R)$ are known only for some special rings R and we refer to [6, §1] for a brief review of recent developments in this particular case.

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