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CONSTRUCTION OF HIGHER GENUS MINIMAL SURFACES WITH ONE END AND FINITE TOTAL CURVATURE

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Abstract. We prove that there exist complete minimal surfaces in the Euclidean 3-space with one Enneper-type end and finite total curvature which have two parameters j, k and are of genus jk, where j and k are positive integers. Our main problem is *the period problem*: each surface has j periods to be killed. We prove that these periods can be killed simultaneously.

Introduction. Recently, many minimal surfaces with higher genus and finite total curvature have been found. Costa [Co] found a complete minimal surface of genus one in $\boldsymbol{R}^3$. Hoffman and Meeks [HM2] proved its embeddedness, and they found higher genus embedded surfaces which are similar to Costa's surface, but with higher-order rotational symmetry. Wohlgemuth [W] proved rigorously the existence of several higher genus minimal surfaces which have embedded ends, including a surface which was found by Hoffman, Meeks and Callahan only numerically. In 1982, Chen-Gackstatter [CG] found surfaces which have one end and are of genera one and two. The genus one C-G surface was generalized by Karcher [K] and the genus two C-G surface was generalized by Thayer [T]. These generalizations are similar to C-G surfaces, but with higher winding order at the end.

Since minimal surfaces in $\boldsymbol{R}^3$ are given by Weierstrass data and path integrals, we always must check well-definedness of the surfaces, and this is called *the period problem*. Chen-Gackstatter [CG] also gave Weierstrass data for a genus three surface, but they did not solve its period problem. Thayer [T] conjectured that the period problem can be solved for arbitrary genus and gave numerical evidence to support this. Espírito-Santo [E] solved the genus three case with a numerical argument.

Rossman suggested to me *a homotopy argument* (which can be thought of as an intermediate value theorem of several variables) for solving the period problem. Wohlgemuth [W] used the homotopy argument to solve period problems for minimal surfaces with four embedded ends.

In this paper, we solve the period problem of the generalized C-G surfaces for arbitrary genus (2.1) by using the intermediate value theorem of several variables.

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