Tôhoku Math. J.
48 (1996), 471–499

ON THE ERGODIC PROPERTIES OF POSITIVE OPERATORS

Dedicated to Professor Satoru Igari on his sixtieth birthday

RYOTARO SATO

(Received February 27, 1995, revised June 12, 1996)

Abstract. In this paper we investigate the ergodic properties of a positive linear operator on a vector lattice of real-valued measurable functions on a sigma-finite measure space. Some results of Ornstein and Brunel are unified and improved.

1. Introduction and definitions. Let $(X, \mathscr{F}, m)$ be a σ-finite measure space and L a vector lattice of real-valued measurable functions on $(X, \mathscr{F}, m)$ under pointwise operations. Thus we understand that if $f \in L$ then the function $f^+(x) = \max\{f(x), 0\}$ is also in L and two functions f and g in L are not distinguished provided that $f(x) = g(x)$ for almost all $x \in X$. Hereafter all statements and relations will be assumed to hold modulo sets of measure zero. Let T be a *positive* linear operator on L. By this we mean that if $f \in L^+$ then $Tf \in L^+$, where L^+ is the cone of nonnegative functions in L. T is said to be *countably additive* if $f_n \in L^+$, $f_n \geq f_{n+1}$ a.e. on X for each $n \geq 1$ and $\lim_n f_n = 0$ a.e. on X then $\lim_n Tf_n = 0$ a.e. on X. A function $\tilde{f}$ in L^+ is called a *modification* of $f \in L^+$ if there exists an $h \in L^+$ such that

$$\tilde{f} = f - h + Th .$$

It is easily seen that for each $n \geq 0$, $T^n f$ is a modification of f. Further, if f_1 is a modification of f and f_2 is a modification of f_1 then f_2 is a modification of f. A convex combination of modifications of f is a modification of f.

Given a $u \in L^+$, we define a subspace $L(u)$ of L by

$$L(u) = \left\{ f \in L : |f| \leq N \sum_{k=0}^{N} T^k u \text{ for some } N \geq 1 \right\}.$$

It is clear that $L(u)$ is a vector lattice and $T(L(u)) \subset L(u)$. When we consider the ergodic ratios

$$R_0^n(f, g) = \frac{\sum_{k=0}^{n} T^k f}{\sum_{k=0}^{n} T^k g} \qquad \text{with} \quad f, g \in L^+ ,$$

1991 *Mathematics Subject Classification*. Primary 47A35.

Key words and phrases: vector lattice, positive linear operator, ergodic ratios and averages, admissible sequences, ratio ergodic theorem, pointwise ergodic theorem.