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# INTERFACIAL PROGRESSIVE WATER WAVES —A SINGULARITY-THEORETIC VIEW

HISASHI OKAMOTO

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**Abstract.** Interfacial water waves of permanent profile between two fluids of different densities are considered. We will show that interfacial waves are generalizations of surface waves, which have been studied extensively in both mathematical and physical papers. The purpose of the present paper is to give a mathematical explanation for numerical results on bifurcations of surface waves by Shōji and ourselves. A hypothesis of degeneracy plays a key role in the present analysis. In fact, we showed in an early paper that a certain degenerate bifurcation point, if it is assumed to be present, can elucidate the complicated bifurcation structure of the surface waves by Shōji. However, in previous papers, we proved unexpectedly that any degenerate bifurcation point does not appear if we vary the depth of the flow. So, the idea of degeneracy has not been physically substantiated in the category of surface waves. In this paper we prove that such a degenerate bifurcation point actually exists when we vary the ratio of the propagation speeds between the upper and lower fluids. Consequently the complicated structure of the surface waves can be explained by regarding the surface waves as special cases of the interfacial waves.

**1. Introduction.** We consider progressive water waves on the interface of two fluids with different densities. By definition, progressive waves move at a constant speed and do not change their profiles during the motion. Therefore the profiles are stationary when we observe them in a suitably moving coordinate system. The interface is a free boundary to be sought. We employ the following terminology:

- *surface wave* means a free boundary and the associated fluid motion in which the fluid below the free boundary is taken into account but the fluid motion above the free boundary is neglected;
- *interfacial wave* means a free boundary and the associated fluid motion in which both the motions above and below the interface are taken into account.

The present paper has two purposes. The first of them is to reformulate the problem of the interfacial waves by modifying Kotchine [13]. We will show that our reformulated interfacial wave problem contains the surface wave problem as a special case. We actually shows that our formulation contains a new parameter: $b=m_u c_u^2/(m_l c_l^2)$, where $m_u$ and $m_l$ are the mass densities of the upper and lower fluids, respectively, and $c_u$ and $c_l$ are mean speeds of the upper and lower fluids, respectively. When the parameter $b$ is equal to zero, our formulation reduces to the surface wave problem given in [16].

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