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BOCHNER-RIESZ MEANS ON SYMMETRIC SPACES

CHRISTOPHER MEANEY AND ELENA PRESTINI

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Abstract. We combine results of Giulini and Mauceri and our earlier work to obtain an almost-everywhere convergence result for the Bochner-Riesz means of the inverse spherical transform of bi-invariant L^p functions on a noncompact rank one Riemannian symmetric space. Following a technique of Kanjin, we show that this result is sharp.

1. Notation. Suppose that G/K is a noncompact rank one Riemannian symmetric space of dimension d. Here functions on G/K can be viewed as being right-K-invariant functions on G, and K-invariant functions on G/K are identified with bi-K-invariant functions on G. Denote by $-\Delta_0$ the Laplace-Beltrami operator on G/K, and $-\Delta$ its self-adjoint extension to $L^2(G/K)$. Its spectral resolution is

$$-\Delta=\int_{|\rho|^2}^{\infty} t dE(t)\,,$$

where the constant $|\rho|^2$ depends on the geometry of G/K. For every $z\in\boldsymbol{C}$ with $\Re(z)\geq 0$ there are the Bochner-Riesz mean operators

$$S_R^z f=\int_{|\rho|^2}^{\infty}\left(1-\frac{t}{R}\right)_+^z dE(t)f\,.$$

In fact there is a $C^\infty(K\backslash G/K)$ kernel s_R^z so that

$$S_R^z f=f*s_R^z\,,\qquad \text{for all}\quad f\in C_c^\infty(G/K)\,.$$

The special case $z=0$ amounts to the usual partial sums:

$$S_R^0 f=E_R f\,,\qquad \text{for}\quad R\geq|\rho|^2\,,$$

and these converge in norm for elements $f\in L^2(G/K)$,

$$\|f-S_R^0\|_2\to 0\,,\qquad \text{as}\quad R\to\infty\,.$$

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