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K-CONTACT RIEMANNIAN MANIFOLDS ISOMETRICALLY IMMERSED IN A SPACE OF CONSTANT CURVATURE

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Introduction. A K-contact Riemannian manifold (M, ξ, g) is a Riemannian manifold (M, g) admitting a unit Killing vector field ξ satisfying

$$R(X, \xi)\xi = g(X, \xi)\xi - X \tag{1.1}$$

where R donotes the Riemannian curvature tensor of (M, g). A K-contact Riemannian manifold is Sasakian, if we have

$$R(X, \xi)Z = g(X, Z)\xi - g(\xi, Z)X . \tag{1.2}$$

In the preceding papers [3] and [4], each of the present authors studied isometric immersions of Sasakian manifolds (M^m, ξ, g) in a space $(^*M^{m+1}, G)$ of constant curvature. Now we show that the results are generalized to K-contact Riemannian manifolds.

THEOREM A. *If a K-contact Riemannian manifold* (M^m, ξ, g) *is isometrically immersed in a space* $(^*M^{m+1}, G)$ *of constant curvature, then* (M^m, ξ, g) *is Sasakian.*

This theorem gives a sufficient condition for a K-contact Riemannian manifold to be Sasakian.

By Theorem A above and the first theorem in [4], we have

THEOREM B. *Let* (M^m, ξ, g) *be a K-contact Riemannian manifold which is isometrically immersed in a space* $(^*M^{m+1}, G)$ *of constant curvature* 1. *Then*
(i) *the type number* $k \leqq 2$, *and*
(ii) (M^m, ξ, g) *is of constant curvature* 1 *if and only if the scalar curvature* $S = m(m-1)$.

By a theorem of B.O'Neill and E.Stiel [1] that a complete Riemannian manifold (M^m, g) of constant curvature $C>0$ which is isometrically immersed in a