Tôhoku Math. Journ.
24 (1972), 331-345.

ON THE ANALYTICITY OF THE KERNEL OF A CLASS OF CONVOLUTION TRANSFORM

Dedicated to Professor Gen-ichirô Sunouchi on his 60th birthday

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(Received Nov. 17, 1971; Revised March 17, 1972)

1. **Introduction.** In the previous papers [6], [7] we have studied the convergence properties and inversion theory of convolution transform

$$f(x) = \int_{-\infty}^{\infty} G(x-t)d\alpha(t) , \tag{1}$$

for which the kernel $G(t)$ is of the form

$$G(t) = \frac{1}{2\pi i}\int_{-i\infty}^{i\infty} [F(s)]^{-1}e^{st}ds . \tag{2}$$

Here $F(s)$ is the meromorphic function with only real zeros and poles, and is of the form

$$F(s) = e^{bs}\prod_{k=1}^{\infty}(1 - s/a_k)e^{s/a_k}/(1 - s/c_k)e^{s/c_k} ,$$

where b, $\{a_k\}_1^\infty$, $\{c_k\}_1^\infty$ are constants such that $0 \leqq a_k/c_k < 1$, $\sum_{k=1}^{\infty} a_k^{-2} < \infty$ and c_k may be equal to $\pm\infty$.

In these papers we assumed the order of $[F(s)]^{-1}$ as $|\tau| \to \infty (s = \sigma + i\tau)$, however, this order should be determined originally by the correlation of zeros a_k and poles c_k of $F(s)$.

From this point of view, Z. Ditzian and A. Jakimovski [1], [2], [3] showed that for all integer $n \leqq N$ ($N \equiv N(\{a_k\}, \{c_k\})$)

$$|F(s)|^{-1} = O(|\tau|^{-n}) \qquad |\tau| \to \infty$$

uniformly in the strip $|\sigma| \leqq R$ for every R and they obtained the inversion formula of the transform (1) which differs from that of ours where it was constructed by repeated integro-differential operators and our formula consisted of integral operator and differential operator separated from each other.

If the series $\sum_{k=1}^{\infty}(a_k^{-1} - c_k^{-1})$ converges then the kernel $G(t)$ becomes a special one called class III kernel and has the characteristic properties ([2], [7]).