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SCORZA-DRAGONI PROPERTY OF FILIPPOV MAPPINGS

Dedicated to Professor Taro Yoshizawa on his sixtieth birthday

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The notion of Filippov's generalized solution of differential equation [1] can be introduced in the following way. A function $x(t)$ is a solution in Filippov's sense of $\dot{x} = f(t, x)$ if $x(t)$ is a solution of differential relation $\dot{x} \in F(t, x)$ where the Filippov mapping F is defined by

$$F(t, x) = \bigcap_{d>0} \bigcap_{l(N)=0} \overline{\mathrm{Conv}}\, f(t,\ U(x, d) - N)$$

where $U(x, d) = \{y: \|y - x\| < d\}$ are regions on an n-dimensional linear normed space R_n, $N \subset U(x, d)$ and l is the Lebesgue measure in R_n.

To the author's best knowledge, the first who pointed out a certain minimality property of the Filippov mapping was Kurzweil who formulated this property in [2] for the autonomous case. In particular, it is proved there that the Filippov mapping $F(x)$ has the smallest possible value for every x among all upper semi-continuous mappings having compact, convex values and containing values of f almost everywhere. The precise formulation of this property will be given later.

It was shown in [3] that the construction of the Filippov mapping can be based solely on the minimal property. Further, it was shown there that in the frame of this new construction we can easily modify the definition in the way that the values of the modified Filippov mapping need not be convex. Generally we assume that $F(t, x)$ takes its values from a given family $\mathscr{E}$ of sets. The family $\mathscr{E}$ may be the family of all compact sets. If, for example, $\mathscr{E}$ is the family of all Cartesian products of intervals, the solutions of the corresponding differential relation are precisely the generalized solutions in Viktorovskii's sense of the original equation [4]. Note that the definition of Viktorovskii is a pure analytical one and the above geometrical characterization of Viktorovskii's solutions is due to Pelant [5], [6].

The aim of the present paper is the investigation of measurability properties of the modified Filippov mappings. Roughly speaking a mapping has the Scorza-Dragoni property if there exists a sequence of cylindrical sets covering the domain of definition up to a set of measure