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AUTOMORPHISM GROUPS, ISOMORPHIC TO $GL(3, F_2)$, OF COMPACT RIEMANN SURFACES

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Let X be a compact Riemann surface of genus $g \geq 2$. The automorphism group $\mathrm{Aut}(X)$ can be represented as a subgroup of $GL(g, C)$, since elements of $\mathrm{Aut}(X)$ act on the g-dimensional module of abelian differentials of X. We denote the representation by $\rho: \mathrm{Aut}(X) \rightarrow GL(g, C)$, and denote the image by $\rho(AG; X)$ for a subgroup AG of $\mathrm{Aut}(X)$. We have studied groups which are $GL(g, C)$-conjugate to $\rho(AG; X)$ for some X with fixed g and some AG. These groups are said to come from a Riemann surface X (see Definition 1). In this connection, we have introduced the CY-, RH- and EX-conditions (see Definitions 2, 3 and 5 in § 1). We saw in [6] that all groups which satisfy the CY- and RH-conditions come from Riemann surfaces except for two groups, i.e., the dihedral group $\mathscr{D}_8$ and the quaternion group $\mathscr{Q}_8$ in the case of $g = 5$. Recently, on the other hand, Kimura [3], [4] studied which groups (isomorphic to $\mathscr{D}_8$, $\mathscr{Q}_8$ or $\mathfrak{U}_5$) come from Riemann surfaces for unspecified g (≥ 2).

In this paper, we consider for unspecified g (≥ 2) the CY- and RH-conditions for groups isomorphic to $GL(3, F_2)$ of 3×3 invertible matrices with entries in the field F_2 with two elements. We take the group $GL(3, F_2)$ since it is the simple Hurwitz group of the smallest order. We apply the character theory of groups and see that if G $(\simeq GL(3, F_2))$ satisfies the CY- and RH-conditions, then G comes from Riemann surfaces except in very few cases. This phenomenon seems to be rooted in some structure of groups although we cannot explicitly point out which.

1. Preliminaries.

DEFINITION 1 (cf. [5], [6]). A subgroup $G \subset GL(g, C)$ is said to come from a compact Riemann surface of genus g, if there exist a compact Riemann surface of genus g and a subgroup AG of $\mathrm{Aut}(X)$ such that $\rho(AG; X)$ is $GL(g, C)$-conjugate to G.

DEFINITION 2 (cf. [5], [6]). $G \subset GL(g, C)$ is said to satisfy the CY-condition if every element of $CY(G) = \{K \mid \text{nontrivial cyclic subgroup of } G\}$ comes from a compact Riemann surface of genus g.

DEFINITION 3 (cf. [8]). Assume that $G \subset GL(g, C)$ satisfies the E-condition, i.e.,

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