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GEOMETRIC FINITENESS, QUASICONFORMAL STABILITY AND SURJECTIVITY OF THE BERS MAP FOR KLEINIAN GROUPS

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1. Introduction. The group Möb of all Möbius transformations acting on the extended complex plane $\hat{C}$ is identified with the 3-dimensional complex Lie group $PSL(2, C)$. A discrete subgroup G of Möb is said to be Kleinian if its region of discontinuity $\Omega(G)$ in $\hat{C}$ is not empty. $\Lambda(G):=\hat{C}-\Omega(G)$ is called the limit set of G. If $\Lambda(G)$ contains infinitely many points, we say G is non-elementary. Throughout this paper, we denote by *G a finitely generated non-elementary Kleinian group* which may contain elliptic elements. For this G, we consider the following three conditions (A), (B) and (C), which are defined later in this section.

(A) G is geometrically finite.
(B) G is quasiconformally (QC) stable.
(C) The Bers map $\beta^*: B(\Omega(G), G)\to PH^1(G, \Pi)$ is surjective.

In §4 and §5 of this paper (Corollaries 1 and 2), we prove

$$(\mathrm{A}) \Rightarrow (\mathrm{B}) \Leftrightarrow (\mathrm{C}).$$

Stronger results for more restricted torsion-free Kleinian groups were obtained by Sullivan [16]. Concerning other known partial solutions to our problem, one may refer to §2.

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(A) Geometric finiteness is the most familiar criterion for Kleinian groups to be "good". G is said to be geometrically finite if the action of G as isometries on the hyperbolic space H^3 has a finite-sided Dirichlet fundamental polyhedron. There is a well-known equivalent characterization by Beardon-Maskit (see [10, Chap. VI. C. 7]).

(B) To define quasiconformal (QC) stability, we choose a system of generators of $G=\langle g_1, \cdots, g_k\rangle$. All our arguments do not depend on the choice of generators. A homomorphism $\chi: G\to$Möb is determined by the images of the generators $(\chi(g_1), \cdots, \chi(g_k))$, which satisfy relations arising from the relations satisfied by $g_1, \cdots, g_k$. In this sense, we represent by χ not only a homomorphism of G but also a point of the product manifold $(\text{Möb})^k$. Therefore the set Hom$(G,$ Möb$)$ of homomorphisms $\chi: G\to$Möb can be regarded as a subvariety of $(\text{Möb})^k$, which is an