RELATIONSHIPS BETWEEN BC-STABILITIES AND p-STABILITIES IN FUNCTIONAL DIFFERENTIAL EQUATIONS WITH INFINITE DELAY

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Abstract. For functional differential equations on a fading memory space, some relationships between the BC-stabilities and p-stabilities are studied. Although the BC-uniform stability is weaker than the p-uniform stability, it is shown that the BC-total stability and BC-uniform asymptotic stability are respectively equivalent to the p-total stability and p-uniform asymptotic stability.

1. Introduction. In the theory of functional differential equations (FDEs) with infinite delay as well as integrodifferential equations, the space BC which consists of all bounded continuous functions on \((-\infty, 0]\) is one of the important classes for the space of initial functions. When one takes the BC as the space of initial functions, there are mainly two ways to provide it with the structure of a metric space in connection with stability problems. One way is to provide it with the supremum norm, and the other is of compact open topology induced by a metric which is called the "p-metric." Throughout this paper, the stabilities corresponding to the two metrics are referred to as the BC-stabilities and the p-stabilities, respectively. Although there are some similarities between them, some authors have studied them independently; for the BC-stabilities, see [2], [3], [8], [9]; for the p-stabilities, see [5]–[7], [11], [13]. Practical phenomena are intimately related to the BC-stabilities. Thus the BC-stabilities would seem to be more usual than the p-stabilities. However, the supremum norm never fade the past memory in contrast with the p-metric. This fact would produce some difficulties when one tries to discuss the existence of periodic solutions or almost periodic solutions under some BC-stability assumption. It is a remarkable difference between the BC-stabilities and p-stabilities. The purpose of this paper is to study the relationships between the above two stabilities. In what follows, we will do this for FDEs considered on a fading memory space. A fading memory space is a considerably flexible (phase) space for FDEs. Indeed, as pointed out in [1], some integrodifferential equations can be set up as FDEs on a fading memory space. Hence our setting is not so restrictive. As will be seen later, the BC-uniform stability does not necessarily imply the p-uniform stability. However, the total stability is an equivalent concept in the BC-stabilities and p-stabilities (Theorem 1). Therefore, via the p-stabilities, one can often overcome some difficulties which would arise in the BC-stabilities. In fact, we

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