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THE LAZY LOGIC OF PARTIAL TERMS

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Abstract. The Logic of Partial Terms **LPT** is a strict negative free logic that provides an economical framework for developing many traditional mathematical theories having partial functions. In these traditional theories, all functions and predicates are strict. For example, if a unary function (predicate) is applied to an undefined argument, the result is undefined (respectively, false). On the other hand, every practical programming language incorporates at least one nonstrict or lazy construct, such as the if-then-else, but nonstrict functions cannot be either primitive or introduced in definitional extensions in **LPT**. Consequently, lazy programming language constructs do not fit the traditional mathematical mold inherent in **LPT**. A nonstrict (positive free) logic is required to handle nonstrict functions and predicates.

Previously developed nonstrict logics are not fully satisfactory because they are verbose in describing strict functions (which predominate in many programming languages), and some logicians find their semantics philosophically unpalatable. The newly developed Lazy Logic of Partial Terms **LL** is as concise as **LPT** in describing strict functions and predicates, and strict and nonstrict functions and predicates can be introduced in definitional extensions of traditional mathematical theories. **LL** is "built on top of" **LPT**, and, like **LPT**, admits only one domain in the semantics. In the semantics, for the case of a nonstrict unary function h in an **LL** theory T, we have $\models_T h(\perp) = y \leftrightarrow \forall x(h(x) = y)$, where $\perp$ is a canonical undefined term. Correspondingly, in the axiomatization, the "indifference" (to the value of the argument) axiom $h(\perp) = y \leftrightarrow \forall x(h(x) = y)$ guarantees a proper fit with the semantics. The price paid for **LL**'s naturalness is that it is tailored for describing a specific area of computer science, program specification and verification, possibly limiting its role in explicating classical mathematical and philosophical subjects.

Free Logic: Aristotle's ambiguous legacy.

> ... not even with these (contraries 'Socrates is well' and 'Socrates is sick') is it necessary always for one to be true and the other false. For if Socrates exists one will be true and the other false, but if he does not both will be false.... (**Categories**, x, 13b12)
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> ...Homer is something (say, a poet). Does it follow that he is? No .... (**De Interpretatione**, xi, 21a26)

Russell's realism.

> A robust sense of reality is necessary in framing a correct analysis of propositions about ... round squares and other such pseudo-objects....we shall insist that in the analysis of propositions, nothing "unreal" is to be admitted. (Bertrand Russell, **Introduction to Mathematical Philosophy**)

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