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GENTZEN'S PROOF OF NORMALIZATION FOR NATURAL DEDUCTION

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Gentzen writes in the published version of his doctoral thesis *Untersuchungen über das logische Schliessen* (Investigations into logical reasoning) that he was able to prove the normalization theorem only for intuitionistic natural deduction, but not for classical. To cover the latter, he developed classical sequent calculus and proved a corresponding theorem, the famous cut elimination result. Its proof was organized so that a cut elimination result for an intuitionistic sequent calculus came out as a special case, namely the one in which the sequents have at most one formula in the right, succedent part. Thus, there was no need for a direct proof of normalization for intuitionistic natural deduction. The only traces of such a proof in the published thesis are some convertibilities, such as when an implication introduction is followed by an implication elimination [1934–35, II.5.13]. It remained to Dag Prawitz in 1965 to work out a proof of normalization. Another, less known proof was given also in 1965 by Andres Raggio.

We found in February 2005 an early handwritten version of Gentzen's thesis, with exactly the above title, but with rather different contents: Most remarkably, it contains a detailed proof of normalization for what became the standard system of natural deduction. The manuscript is located in the Paul Bernays collection at the ETH-Zurich with the signum *Hs.* 974: 271. Bernays must have gotten it well before the time of his being expelled from Göttingen on the basis of the racial laws in April 1933. He seems to have never mentioned the existence of a proof of normalization for natural deduction by Gentzen, even if he discussed Gentzen's work extensively [Bernays 1965, 1970] and was also fully aware of the published proof of normalization by Prawitz [1965]. This is even the more strange if we consider that Bernays had presented Gentzen's calculus of natural deduction at The Institute for Advanced Study in his lectures from 1935–36, titled *Logical Calculus*. These lectures give no indication of Gentzen's aims in setting up the system of natural deduction, such as a subformula property of normal derivations to be used in proofs of consistency.

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