conditions, are classically equivalent to the classes Π_n, and $\bigcup_{n>1} \Phi_n$ exhausts all arithmetical formulas. Moreover, Burr shows that the induction schema restricted to the class Φ_n proves the same Π_2-sentences as the classical fragment of PA defined by the Π_n-induction schema, $I\Pi_n$. In other words, the Visser–Wehmeier theorem does not hold for the classes Φ_n. For his result, Burr uses an interesting variation of the so-called Friedman–Dragalin translation due to T. Coquand and M. Hofmann. In my opinion, the isolation of the classes Φ_n is a very basic and important contribution of the paper to intuitionistic arithmetic.

It is worth noticing that the question of the intuitionistic analogues of the classes Σ_n is left open. In fact, even for the classes Π_n, it is not excluded at present that there can be more than one natural counterpart of these classes in intuitionistic arithmetic. Only further research and "experimentation" can settle this question. By Burr's results, however, we now have at least one good candidate.

In the remaining part of the paper some additional facts are established. The next result is an improvement of the Visser–Wehmeier theorem. One defines another hierarchy of classes of formulas Θ_n, essentially by closing the classes Φ_n under existential (as well as universal) quantification. This makes the class Θ_2 already contain all prenex formulas. Burr shows, by means of Gödel's Dialectica interpretation, that the restriction of the induction schema to the class Θ_n proves the same Π_2-sentences as the classical fragment $I\Pi_n$. For $n = 2$ this statement implies the Visser–Wehmeier theorem.

Finally, collection principles in intuitionistic arithmetic are considered. For the standard formulation of the collection principle

$$\forall x \leqq a\, \exists y\, \varphi(x, y) \;\rightarrow\; \exists z\, \forall x \leqq a\, \exists y \leqq z\, \varphi(x, y),$$

where φ is any arithmetical formula, the author shows, using a result of U. Kohlenbach, that the provably total functions are bounded by polynomials. This contrasts with the behavior of this schema within classical logic: classically, it is equivalent to the full induction. The author also verifies that the contrapositive formulation of the collection schema over intuitionistic logic proves the law of the excluded middle, that is, it implies not only HA, but the whole of PA.

The paper is written in a nice and clear style. Although it does not develop significant new "methods," it does introduce an important new definition. It will predictably be often quoted in the future papers on intuitionistic arithmetic.

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Thomas Strahm. *Polynomial time operations in explicit mathematics*. ***The journal of symbolic logic***, vol. 62 (1997), pp. 575–594.

Andrea Cantini. *Feasible operations and applicative theories based on $\lambda\eta$*. ***Mathematical logic quarterly***, vol. 46 (2000), pp. 291–312.

More than two decades ago, Solomon Feferman (*A language and axioms for explicit mathematics*, JSL XLIX 308) introduced systems of explicit mathematics, a feature of which is a core of axioms for the *partial* combinatory logic of self-applicative operations. Feferman systems were originally motivated by foundational issues, and they typically included primitive recursive arithmetic. Influenced by the seminal work of Samuel Buss (***Bounded arithmetic***, JSL LVI 759), Thomas Strahm's paper defines two related systems of explicit mathematics, with a special unary predicate $\mathbb{W}$ for the binary words, whose provably total functions are exactly the polynomial-time computable functions. The main burden of the article lies in reducing (in the spirit of reductive proof theory) these systems to a known feasible theory, i.e., to a theory whose provably total functions are the polynomial-time computable functions. The chosen theory is Ferreira's Σ^b_1-NIA (a variant of Buss's S^1_2) augmented by the scheme of