The existence of pcf generators for a set A of regular cardinals is first proved under the assumption that $2^{|A|} < \min A$ and then with the standard assumption $|A| < \min A$, but the existence of transitive generators, and consequently localization and the equation $\mathrm{PCF}\,\mathrm{PCF}\,A = \mathrm{PCF}\,A$, are proved only under assumptions that are not optimal.

The style of presentation is more that of lecture-notes than of a survey paper. Apart from the repetition in having two proofs for the existence of generators, there are instances where definitions are made within the statement of a lemma, and some proofs are not quite polished.

PCF theory has continued to develop rapidly in the 1990's and therefore many important notions—such as that of a good scale—and important theorems—such as the asymptotic version of GCH in ZFC (Saharon Shelah, *The generalized continuum hypothesis revisited*, ***Israel journal of mathematics***, vol. 116 (2000), pp. 285–321) and its consequence that for sufficiently large λ, $\Diamond_{\lambda^+}$ holds iff $2^\lambda = \lambda^+$—of course are not covered by this paper. The *Analytical guide* and the *Annotated content of continuations* sections at the end of ***Cardinal arithmetic*** are recommended to readers who wish to check what is known in the field (or rather, what was known about 1994). Another newer source of pcf theory is Holz, Steffens, and Weitz, ***Introduction to cardinal arithmetic*** (Birkhäuser, 1999), which is partially based on the paper under review and which contains additional material.

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THOMAS JECH. *Singular cardinal problem: Shelah's theorem on $2^{\aleph_\omega}$*. ***Bulletin of the London Mathematical Society***, vol. 24 (1992), pp. 127–139.

In this paper Tom Jech presents, in his familiar readable style, a self-contained proof in about ten pages of Shelah's

THEOREM 1. $[(\forall n)(2^{\aleph_n} < \aleph_\omega)] \Rightarrow 2^{\aleph_\omega} < \aleph_{\omega_4}$

The secret of the paper's brevity is the repeated use it makes of the assumption $2^{\aleph_0} < \aleph_\omega$ in proving particular instances of pcf theorems—theorems that hold also *without* this assumption, indeed without any assumptions at all beyond ZFC. The use of the additional assumptions allows a considerable simplification of the proofs (for example, the existence of least upper bounds modulo ideals can be gotten from the Erdős–Rado theorem easily, but needs a lengthy argument otherwise).

Shelah proved in fact a stronger theorem:

THEOREM 2. $\mathrm{cf}([\aleph_\omega]^{\aleph_0}, \subseteq) < \aleph_{\omega_4}$

This theorem is stronger because it makes no assumptions on the size of the continuum; in the (consistent) case that the continuum is equal to $\aleph_{\omega_5+23}$, it is still true, by this theorem, that the least number of countable subsets of $\aleph_\omega$ required to cover *all* countable subsets of $\aleph_\omega$ is smaller than $\aleph_{\omega_4}$, although each countable set contains more than $\aleph_{\omega_4}$ subsets.

Shelah's work on cardinal arithmetic made it clear that in ZFC itself, many good theorems about *revised* singular cardinal arithmetic can be proved—such as the one above, which gives an absolute upper bound on the *cofinality* of $[\aleph_\omega]^{\aleph_0}$, rather than on its cardinality (which would be of course impossible, by Cohen's proof). So in this respect there is a major point lost in the presentation made in this paper, and it may be a bit misleading in not emphasizing this innovative aspect of Shelah's contribution to cardinal arithmetic.

In this paper the author does a great job in presenting the first theorem above in minimum space; but a reader who would like to know why there exists a Jónsson algebra on $\aleph_{\omega+1}$—or why the *second* theorem above is true—would not be able to use any of the proofs in it.

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