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GROUP PRESENTATION OF THE SCHUR-MULTIPLIER DERIVED FROM A LOOP GROUP

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1. Introduction

In 1960s H. Matsumoto [2] considered the universal central extension and the Schur-multiplier of a Chevalley group which is derived from an arbitrary field F and an arbitrary Cartan matrix A of finite type. Then he showed that the corresponding Steinberg group (we denote it by $St(A,F)$) is its universal central extension and gave a presentation of its Schur-multiplier for almost every field. Now one sees this Schur-multiplier is an abelian group which is strongly connected with this root system.

In general, a Chevalley group $G(A,R)$ over a commutative ring R is constructed as a group using the functor represented by some Hopf algebra. And there are many results about the structure of the associated K_2 group.

In this paper we take Laurent polynomial rings $F[X,X^{-1}]$. A Chevalley group over a Laurent polynomial ring is sometimes called a loop group. Then we consider the structure of the K_2 group of a loop group and obtain the following theorem, where $\hat{K}_2$ will be given by generators and relations in section 3.1.2.

Theorem

Let A be a Cartan matrix of finite type. Then we have

$$K_2(A,F[X,X^{-1}]) \simeq \widehat{K_2}(A,F[X,X^{-1}]).$$

2. Preliminaries

In this section K is a field of characteristic 0. Let $X = (X_{ij})$ $(1 \le i,j \le n)$ be an $n \times n$ symmetrizable generalized Cartan matrix. We denote a Kac-Moody Lie algebra over K, the standard Cartan subalgebra, the associated root system, the set of real roots obtained from X, by $\mathfrak{g}(X)$, $\mathfrak{h}$, Δ, Δ^{re} respectively. Using this notation we can decompose $\mathfrak{g}(X)$ as follows:

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