TSUKUBA J. MATH.
Vol. 30 No. 1 (2006), 31–47

ON THE BRANCHING THEOREM OF THE PAIR $(F_4, Spin(9))$

By

Katsuya MASHIMO

Introduction

Let G be a compact connected Lie group and K be a closed subgroup. A finite dimensional complex irreducible representation $V^G(\lambda)$ of G with highest weight λ is decomposed into a direct sum of irreducible representations $V^K(\mu)$ of K with highest weight μ

$$V^G(\lambda) = \sum_{\mu} m(\lambda, \mu) V^K(\mu).$$

Let $V = G \times_K V^K(\mu)$ be the irreducible complex homogeneous vector bundle on $M = G/K$. By a theorem of Peter and Weyl, the space of sections $\Gamma(V)$ of V is a unitary direct sum of finite dimensional representations of G. By the Frobenius reciprocity theorem, the multiplicity of a complex irreducible representation $V^G(\lambda)$ in $\Gamma(V)$ conincides with the coefficient $m(\lambda, \mu)$.

Branching theorem of the pair $(F_4, Spin(9))$ was studied first by Lepowsky ([1], [2]). His result is not sufficient to decompose the space of sections $\Gamma(V)$, for the main interest of Lepowsky's work is in those pairs (λ, μ) with $m(\lambda, \mu) = 1$ (see also [3]). In the previous paper [4], the author carried the Lepowsky's calculation forward for the purpose of giving the decomposition of the space of complex p-form on the Cayley projective plane $P^2(\mathbf{Ca})$. Actually we obtained the decomposition for $p \leq 5$ and applied them to calculate the spectra of Laplacian Δ^p acting on p-forms of $P^2(\mathbf{Ca})$.

In [5], F. Sato studied the stability of branching coefficient. Roughly speaking, the branching coefficnent $m(\lambda, \mu)$ satisfies $m(\lambda, \mu) = m(\lambda + \lambda_0, \mu)$ if λ_0 is a spherical representation of (G, K) and λ is sufficiently large.

In this note we will prove the following

Received June 15, 2004.
Revised October 11, 2005.