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HOPF ALGEBRAS GENERATED BY A COALGEBRA

By

Charles B. RAGOZZINE, Jr.

Abstract. The concept of a free Hopf algebra generated by a coalgebra was introduced by Takeuchi to provide an example of a Hopf algebra with a non-bijective antipode. In general, this free Hopf algebra is not generated as an algebra by the coalgebra. In this paper, we construct a class of Hopf algebras, including $SL_q(2)$, which are generated as algebras by a coalgebra and which satisfy a useful universality condition.

Introduction

The paper is presented in three parts. First, a class of Hopf algebras which are generated as algebras by a coalgebra is constructed. Next, the universality of this class of Hopf algebras is addressed. Finally, relevant examples to this discussion are considered, including $SL_q(2)$.

Most of the important preliminaries can be found in [1] and [2]. In particular, following [1], we will use the superscripts "*op*" and "*cop*" to refer to the opposite algebra and opposite coalgebra, respectively. We will also make use of the well-known fact that the tensor algebra of a coalgebra (C, Δ, ε), denoted $(T(C), \bar{\mu}, \bar{\eta}, \bar{\Delta}, \bar{\varepsilon})$, is a bialgebra. For a reference, see [3].

1. The Construction

LEMMA 1.1. *Suppose that (C, Δ, ε) is a coalgebra, $(B, \mu_B, \eta_B, \Delta_B, \varepsilon_B)$ is a bialgebra, and $f : C \to B$ is a coalgebra map. Then, there exists a unique bialgebra map $\bar{f} : T(C) \to B$ extending f.*

PROOF. By the universality of $T(C)$, we know that f induces a unique algebra map $\bar{f} : T(C) \to B$. It remains to show that $\bar{f}$ is a coalgebra map, which requires $\varepsilon_B \circ \bar{f} = \bar{\varepsilon}$ and $\bar{f} \otimes \bar{f} \circ \bar{\Delta} = \Delta_B \circ \bar{f}$. Identify C with its image in $T(C)$,

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