TSUKUBA J. MATH.
Vol. 25 No. 2 (2001), 279–298

SOME DIFFERENTIAL-GEOMETRIC PROPERTIES OF R-SPACES

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§ 0. Introduction

Let G/K be an irreducible Riemannian symmetric space, where G is a connected compact semisimple Lie group and K its closed subgroup. The adjoint representation group $\mathrm{Ad}(K)$ acts on the tangent space $T_o(G/K)$ of G/K at the origin o as an isometry group. Let S denote a unit hypersphere in the $T_o(G/K)$ centered at the origin o. For each point a of S, the orbit $\mathrm{Ad}(K)a$ of a under $\mathrm{Ad}(K)$ is called an *R-space*. The R-spaces form an abundant class of homogeneous Riemannian manifolds and have several distinguished properties as submanifolds of S, and so they have been investigated by many authors from the point of view of differential geometry. (e.g., [5], [10], [12], [13], [16], [17], [21], [22], [24], [31], [32], [33])

In this paper, for these R-spaces we shall study the following:

(I) In the case where G/K is Hermitian, we investigate some relations between the complex structure and the restricted root system with respect to G/K.

(II) We express the covariant derivative of the second fundamental form of every R-space in S with respect to the Lie brackets in the Lie algebra of G.

As an application of (I), we obtain many new examples of homogeneous CR-submanifold in a complex projective space, which is stated as Theorem 3.2. As an application of (II), we can give a partial solution to the S. Maeda's Problem, which is stated as Corollary 4.5.

The author would like to express her thanks to Professor R. Takagi for his valuable suggestions and constant encouragements.

§ 1. Preliminaries

In this paper, let G/K be an irreducible Riemannian symmetric space of compact type once and for all, where G is a connected compact semisimple Lie group

Received March 10, 2000.
Revised October 26, 2000.