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INTEGRAL GEOMETRY OF REAL SURFACES IN COMPLEX PROJECTIVE SPACES

Dedicated to Professor Katsuhiro Shiohama on his sixtieth birthday

By

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1. Introduction

The purpose of this paper is to give a Poincaré formula of real surfaces in complex projective spaces stated in the following theorem.

THEOREM 1.1. *Let CP^n be a complex projective space of complex dimension n, M a real submanifold of CP^n of real dimension 2 and N a complex submanifold of complex dimension $n-1$. Then we have*

$$\int_{U(n+1)} \sharp(M \cap gN)\, d\mu_{U(n+1)}(g) = \frac{\operatorname{vol}(U(n+1))\operatorname{vol}(N)}{2\operatorname{vol}(CP^1)\operatorname{vol}(CP^{n-1})}\int_M (1+\cos^2\theta_x)\, d\mu_M(x),$$

where θ_x is the Kähler angle of M at x. Moreover the above formula holds for a real submanifold M of real dimension $2n-2$ and a complex submanifold N of complex dimension 1, where θ_x is the Kähler angle of $T_x^{\perp}M$.

One of the oldest results in integral geometry is the Poincaré formula for the average of the intersection number of two curves. Many differential geometers have studied the Poincaré formula from various points of view. In particular, R. Howard [1] has generalized this formula in Riemannian homogeneous spaces and obtained the following formula.

THEOREM 1.2. [1] *Let G/K be a Riemannian homogeneous space with a G-invariant Riemannian metric and take submanifolds M and N of G/K. Assume that* $\dim M + \dim N = \dim(G/K)$ *and that G is unimodular. Then*

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