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INVARIANT HOMOGENEOUS STRUCTURES ON HOMOGENEOUS REAL HYPERSURFACES IN A COMPLEX PROJECTIVE SPACE AND AN ODD-DIMENSIONAL SPHERE

By

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1. Introduction

In Riemannian geometry the theory of homogeneous spaces is a very interesting subject. Many geometers investigate homogeneous submanifolds in a complex projective space CP_n and get many fruitful results. CP_n has good geometric structures. One of them is a Kähler structure. These structures induce many geometric structures on submanifolds. For example, almost contact metric structures on real hypersurfaces are induced from the Kähler structure of CP_n. These structures are very useful to investigate geometries of real hypersurfaces. On the other hand, CP_n has the Hopf fibration whose total space is the odd-dimensional unit sphere S^{2n+1}. Its projection is a Riemannian submersion. The fundamental equations of Riemannian submersions are investigated by O'Neill [9]. The Hopf fibration is a useful tool when we study geometries of submanifolds in CP_n. Through the Hopf fibration informations of submanifolds in CP_n can be translated into informations of submanifolds in S^{2n+1} and vice versa. Using this method, R. Takagi [11] classified homogeneous real hypersurfaces in CP_n. By his theorem they are classified into 5 types of Riemannian submanifolds, say of type (A)–(E) (see §2 Theorem T).

The homogeneity of a Riemannian manifold can be studied by means of the existence of a so called homogeneous structure tensor (cf. [1] and [14]). So it is natural to expect that on each homogeneous manifold a homogeneous structure tensor will contain geometric informations about this space. Therefore it is an important problem to determine homogeneous structure tensors on homogeneous spaces. In the paper [6] the author gives a homogeneous structure on a homogeneous real hypersurface of type (A) (cf. §4). Using this tensor, we know that

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