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AREA-MINIMIZING OF THE CONE OVER SYMMETRIC *R*-SPACES

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Introduction

Let B denote a submanifold of the unit sphere in $\boldsymbol{R}^n$ and C_B the cone over B, which is the union of rays starting from the origin and passing through B.

A cone is called area-minimizing if the truncated cone C_B^1 inside the unit ball is area-minimizing among all surfaces with boundary B. The surfaces we will use are integral currents. A tangent cone to surface S at a point $p \in S$ can be thought of as the union of rays starting from p and tangent to S at p. This is the generalization of the notion of tangent plane. If the tangent cone at p is not a plane, then p is a singular point of S. If S is area-minimizing, then each tangent cone to S is area-minimizing. Thus in order to study area-minimizing surface with singularities, we need to know which cones are area-minimizing.

G. R. Lawlor proposed a criterion for area-minimization in [5]. His principal idea is to construct an area-nonincreasing retraction $\Pi : \boldsymbol{R}^n \to C$. If S is another surface which has the same boundary as C_B^1, it will follow that

$$\mathrm{vol}(S) \geq \mathrm{vol}(\Pi(S)) \geq \mathrm{vol}(C_B^1)$$

since $\Pi(S)$ must cover all of C_B^1. Using this method, he gave a complete classification of area-minimizing cones C over products of spheres and the first example of minimizing cone over a nonorientable manifold. In order to construct the retraction he solved a differential equation with numerical analysis.

In this paper, we consider the canonical imbeddings of symmetric R-spaces which are linear isotropy orbits of symmetric pairs. Using root systems, we construct area-nonincreasing retractions concretely.

In section 1 we prepare some notation and terminology, and prove an essential theorem (Theorem 1.6) for construction of the retractions. In section 2 we describe the canonical imbeddings of symmetric R-spaces, and construct

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