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SEQUENT CALCULI FOR THREE-VALUED LOGICS

Dedicated for the memory of the late professor Toshio Nishimura

By

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§ 0. Introduction

The purpose of this paper is to give sequent calculi for some 3-valued (propositional) logics in a rather uniform way. Three-valued logic is an old subject that has recently been taken a revived interest in, for its own sake as well as for its potential applications in several areas of computer science.

After general preliminaries in the first section, we deal in § 2 with the 3-valued weakly-intuitionistic logic I^1 introduced in Sette-Carnielli [15]. This logic has one designated value, and its connectives have simple truth-value functions. Sequent calculi for similar logics have been given by Miyama [9], in which Gill's 3-valued predicate logic studied in [5] is concerned with, and by Avron [1], in which Kleene's strong 3-valued logic (Kleene [7, § 64]) is handled.

In § 3, Sette's 3-valued paraconsistent logic P^1 (Sette [14]) is dealt with, which had been introduced in da Costa [3] for underivability proof. This logic has two designated values. Avron [1] has given a sequent calculus for such a logic too, precisely, the 3-valued logic of D'Ottaviano-da Costa [4].

Meanwhile, Wroński's 3-valued logic constitutes the subject of § 4. This logic has one designated value, but the truth-value function of its single connective is rather complicated. Wroński showed in [17] that this logic forms a negative answer to Bloom's problem posed in [2], which asks whether the consequence operation determined by a finite matrix is always finitely based. We will give a sequent calculus for this logic, but this does not conflict with the above result; for, not all the beginning sequents of our calculus are the ones with single succedent formula (cf. 4.2). Meanwhile, this logic has been proved to be finitely axiomatizable in Wojtylak [16]. (According to Palasińska [12], the latter had been proved by Rautenberg [13, p. 116], but unfortunately I could not consult

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