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GENERALIZED HELICAL IMMERSIONS

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Introduction.

In this paper, we assume that all geodesics are parametrized by the arclength. Let f be an isometric immersion of a Riemannian manifold M into a Riemannian manifold $\tilde{M}$. If geodesics in M are viewed as specific curves in $\tilde{M}$, what are the shape of $f(M)$? Several geometricians studied this problem. K. Sakamoto characterized an isometric immersion f of a complete connected Riemannian manifold M into a Euclidean space or a sphere such that every geodesic in M is viewed as a helix in the ambient space and that the order and the Frenet curvatures of the helix are independent of the choice of the geodesic (cf. [15], [16]). In [5], D. Ferus and S. Schirrmacher investigated an isometric immersion f of a compact connected Riemannian manifold M into a Euclidean space $\boldsymbol{R}^m$ satisfying the following condition:

(A) Almost every geodesic in M is viewed as a generic helix in $\boldsymbol{R}^m$.

Here "almost every geodesic" means that the tangent vectors of such geodesics fill the unit tangent bundle of M up to a closed set of measure zero and a generic helix means a helix of even order such that the closure of the image coincides with the lowest dimensional Clifford torus containing it. In [4] and [5], they showed that the condition (A) is equivalent to the following two conditions, respectively:

(B) f is extrinsic symmetric in the sense of [4].

(C) The second fundamental form of f is parallel.

In this paper, we consider an isometric immersion f of a Riemannian manifold M into a Riemannian manifold $\tilde{M}$ such that every geodesic in M is viewed as a helix in $\tilde{M}$, where the order of the helix may depend on the choice of the geodesic. We call such a immersion a *generalized helical immersion* and the highest order of those helices the *order of* f. First, we show that all isometric immersions with parallel second fundamental form are generalized helical. Conversely, it is very interesting to investigate in what case a generalized helical immersion has the

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