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ON SPACES WITH LINEARLY HOMEOMORPHIC FUNCTION SPACES IN THE COMPACT OPEN TOPOLOGY

Dedicated to Professor Akihiro Okuyama on his sixtieth birthday

By

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1. Introduction

For a space X, let $C(X)$ be the linear space of all real-valued continuous functions on X, and let $C_0(X)$ (resp. $C_p(X)$) denote the linear topological space $C(X)$ with the compact-open (resp. pointwise convergence) topology. We say that spaces X and Y are *l_0-equivalent* (resp. *l_p-equivalent*) if $C_0(X)$ and $C_0(Y)$ (resp. $C_p(X)$ and $C_p(Y)$) are linearly homeomorphic. For an ordinal number α, let $X^{(\alpha)}$ be the α-th derived set of a space X, where $X^{(0)} = X$. Recall from [3] that an ordinal α is *prime* if it satisfies the following condition: If $\alpha = \beta + \gamma$, then $\gamma = 0$ or $\gamma = \alpha$. Note that 0 and 1 are only finite prime ordinals. For $\alpha \geq \omega$, α is prime if and only if there is an ordinal $\mu \geq 1$ such that $\alpha = \omega^{\mu}$ (cf. [3, Theorem 2.1.21]). Thus, $\omega, \omega^2, \omega^3, \dots$ and the first uncountable ordinal ω_1 are prime. The purpose of this paper is to improve some theorems in Baars and de Groot [3] by proving the following theorem:

THEOREM 1. *Let X and Y be l_0-equivalent metric spaces. For each prime ordinal $\alpha \leq \omega_1$, we have*:

(a) $X^{(\alpha)} = \varnothing$ *if and only if* $Y^{(\alpha)} = \varnothing$,
(b) $X^{(\alpha)}$ *is compact if and only if* $Y^{(\alpha)}$ *is compact*,
(c) $X^{(\alpha)}$ *is locally compact if and only if* $Y^{(\alpha)}$ *is locally compact.*

Baars and de Groot proved (a), (b) and (c) in Theorem 1 for $\alpha = 0, 1$ under the additional assumption that X and Y are 0-dimensional and separable ([3, Theorems 4.5.2 and 4.5.3]). For l_p-equivalent metric spaces X and Y, they proved (a) for each prime $\alpha \leq \omega_1$ ([3, Theorems 4.1.15 and 4.1.17]), and proved (b) and

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