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ON SELF-INJECTIVE DIMENSIONS OF ARTINIAN RINGS

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Throughout this note R stands for a left and right artinian ring unless specified otherwise. We denote by $\operatorname{mod} R$ (resp. $\operatorname{mod} R^{\mathrm{op}}$) the category of all finitely generated left (resp. right) R-modules and by $(\ )^*$ both the R-dual functors. For an $X \in \operatorname{mod} R$, we denote by $\varepsilon_X : X \to X^{**}$ the usual evaluation map, by $E(X)$ its injective envelope and by $[X]$ its image in $K_0(\operatorname{mod} R)$, the Grothendieck group of $\operatorname{mod} R$.

In this note, we ask when $\operatorname{inj\,dim}{}_R R = \operatorname{inj\,dim} R_R$. Note that if $\operatorname{inj\,dim}{}_R R < \infty$ and $\operatorname{inj\,dim} R_R < \infty$ then by Zaks [10, Lemma A] $\operatorname{inj\,dim}{}_R R = \operatorname{inj\,dim} R_R$. So we ask when $\operatorname{inj\,dim} R_R < \infty$ implies $\operatorname{inj\,dim}{}_R R < \infty$. There has not been given any example of R with $\operatorname{inj\,dim}{}_R R \neq \operatorname{inj\,dim} R_R$. However, we know only a little about the question. By Eilenberg and Nakayama [5, Theorem 18], ${}_R R$ is injective if and only if so is R_R. In case R is an artin algebra, we know from the theory of tilting modules that $\operatorname{inj\,dim}{}_R R \leqq 1$ if and only if $\operatorname{inj\,dim} R_R \leqq 1$ (see Bongartz [3, Theorem 2.1]). Also, if R is of finite representation type, it is well known and easily checked that $\operatorname{inj\,dim}{}_R R < \infty$ if and only if $\operatorname{inj\,dim} R_R < \infty$.

Suppose $\operatorname{inj\,dim} R_R < \infty$. Then we have a well defined linear map

$$\delta : K_0(\operatorname{mod} R^{\mathrm{op}}) \longrightarrow K_0(\operatorname{mod} R)$$

such that

$$\delta([M]) = \sum_{i \geqq 0} (-1)^i [\operatorname{Ext}_R^i(M, R)]$$

for $M \in \operatorname{mod} R^{\mathrm{op}}$. Since R is artinian, both $K_0(\operatorname{mod} R^{\mathrm{op}})$ and $K_0(\operatorname{mod} R)$ are finitely generated free abelian groups of the same rank. Also, for an $M \in \operatorname{mod} R^{\mathrm{op}}$, $[M]=0$ if and only if $M=0$. Thus $\operatorname{inj\,dim}{}_R R < \infty$ if (and only if) the following two conditions are satisfied:

(a) δ is surjective.
(b) There is an integer $d \geqq 1$ such that $\delta([\operatorname{Ext}_R^d(X, R)]) = 0$ for all $X \in \operatorname{mod} R$.

In this note, along the principle above, we will prove the following

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