TSUKUBA J. MATH.
Vol. 15 No. 1 (1991), 57—78

ON THE RESIDUAL TRANSCEDENTAL EXTENSIONS OF A VALUATION. KEY POLYNOMIALS AND AUGMENTED VALUATION

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Let K be a field and v a valuation on K. The problem of extending v to $K(X)$ (the field of rational functions of one inderminate) has been previously considered in some works as [7] and [10]. Particularly in [7], MacLane studied the case when v is discrete and rank one. In solving the problem in this case, MacLane used some notions as key polynomial and augmented valuation.

An extension w of v to $K(X)$ is called residual transcendental (briefly, an r.t. extension) if the residue field of w is a transcendental extension of the residue field of v (MacLane called these extensions "inductive value"). Some aspects of r.t. extensions have been considered in [5, Ch. VI], [9], [1], [2], [3] and [11]. Particularly in [2] and [11] all r.t. extensions of v to $K(X)$ were described using the notion of "minimal pair" (see definition in Section 1). Although in [3] some results on minimal pairs were given, the problem of finding minimal pairs in the general setting seems to be difficult.

In this work we follow, for arbitrary r.t. extensions, MacLane's ideas of key polynomial and augmented valuation and show that these give a powerful tool in the study of all extensions of v to $K(X)$. In particular, the key polynomials over an r.t. extension give us the possibility of defining some new minimal pairs (Theorem 5.1).

Now we briefly describe the content of the paper. Section 1 contains notation, definitions and the main results from [2] and [11], Theorem 1.2 and some consequences of this theorem will be used in this paper.

In section 2, we give some technical results related to the domination of valuations on $K(X)$, which was also introduced by MacLane in [7]. This notion has been used in [4] to describe all valuations on $K(X)$). In Section 3 (after MacLane [7]) key polynomial and augmented valuation are defined.

The key polynomials over an r.t. extension are studied in Section 4. The main results are given in Theorems 4.4 and 4.6. We remark that Theorem 4.6

Received September 25, 1989, Revised September 13, 1990.