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ON AXIOM SCHEMATA APPLICABLE TO THE FORMULAE WITH ε-SYMBOLS

(Dedicated to Prof. S. Maehara for his sixtieth birthday)

By

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§1. Introduction.

As is well known, if a theory T is consistent in the first order predicate calculus, then it is still consistent in the first order predicate calculus with Hilbert's ε-symbol (assumed with equality axioms), which we shall call ε-calculus in short. This fact was given by Hilbert and Bernays [2]. Maehara [3] showed that it is still consistent in ε-calculus extended with additional axioms:

$$(*)\quad \forall x(A(x) \longleftrightarrow B(x)) \longrightarrow \varepsilon x A(x)=\varepsilon x B(x),$$

where $A(a)$ and $B(a)$ are arbitrary formulae with possibly ε-symbols. These theorems however require careful reading, when the theory T has axiom schemata. In applying the theorems, axiom schemata for T may not be applied to formulae containing ε-symbols even in the ε-calculus. For instance, $ZF+\neg AC$, Zermelo-Fraenkel set theroy with the negation of the axiom of choice, yields a contradiction in ε-calculus if the schemata are allowed to apply to all formulae of the language of ε-calculus.

Noting this, we naturally ask the following:

(**) *What kind of axiom schemata can be consistently extended to the ε-calculus in the above sense?*

Even in the model theoretical sense, this is not a trivial question. It is partially answered in [3], where the schema is mathematical induction and the theory in question have certain strong properties. Our purpose here is to answer it in more general cases. The precise description of our result is given in the next section after defining some necessary notions.

§2. Preliminaries.

We use the letters L, L' to denote languages for the first order predicate

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