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KILLING VECTOR FIELDS ON SEMIRIEMANNIAN MANIFOLDS

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Abstract It is well known that a Killing vector field on a riemannian compact manifold is holonomic (Kostant (4)). In other words, the A_X operator ($A_X=L_X-\nabla_X=-\nabla X$) lies in the holomony algebra of the manifold.

The covariant derivative of A_X gives us a curvature transformation. This fact and the Ambrose-Singer theorem show that the A_X operator lies infinitesimally in the holonomy algebra $\boldsymbol{h}$.

$$(\text{i.e. } \forall Y,\ \nabla_Y A_X=R_{XY}\in\boldsymbol{h}) \qquad (*)$$

The subject of our study is the holonomicity of a Killing vector field on a semiriemannian compact manifold. We remark the validity of (*) on semiriemannian manifolds.

In order to simplify its study, we constrain it to Lorentz locally strictly weakly irreducible manifolds (l. SWI). We remark that Berger (1) showed that the holonomy algebra of a Lorentz manifold which is irreducible and non locally symmetric is the whole $po(n, 1)$. Therefore, we can leave out this case.

Strictly weakly irreducible manifolds, defined by H. Wu (5, 6) in 1963 are the cornerstones of this study. Among these we have found examples of compact manifolds with a non holonomic Killing vector field.

0. Preliminaries.

Let M be a semiriemannian manifold of dimension n and signature s and take $p\in M$. Any loop σ with base point p provides us with an isometry of T_pM. The set of isometries can be structured as a Lie group: the holonomy group with base-point p, $G_p(M)$. When we consider only nulhomotopes loops,

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