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HYPERSURFACES IN THE QUATERNIONS II

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1. Introduction.

Let $\mathbf{H}=\mathrm{span}_{\mathbf{R}}\{1, i, j, k\}$ be the quaternions. We shall fix the basis $\{1, i, j, k\}$ throughout this paper. Then, we may regard $\mathbf{H}$ as a 4-dimensional Euclidean space $\mathbf{R}^4$ in the natural way. An oriented hypersurface M^3 in $\mathbf{H}$ admits a global orthonormal frame field as follows. Let (M^3, f) be an oriented hypersurface of $\mathbf{H}$ and ξ a unit normal vector field on M^3. Then $\{\xi i, \xi j, \xi k\}$ is a global orthonormal frame field on $f(M^3)$. We shall call this orthonormal frame field an associated one of $f(M^3)$ and the dual frame field of $\{\xi i, \xi j, \xi k\}$ as associated dual frame field, respectively. We may remark that the associated frame field on M^3 (intrinsically) of an oriented hypersurface (M^3, af) in $\mathbf{H}$ coincides with the associated one of the hypersurface (M^3, f) for any $a\in Sp(1)$. We note that the associated frame field of (M^3, bf) are different from the associated one of (M^3, f) for $b\in SO(4)$ and $b\notin Sp(1)$. This paper is a continuation of the previous one ([3]). Let x be the unit normal vector field of a unit 3-sphere S^3 in $\mathbf{H}$, then the vector fields $\{xi, xj, xk\}$ are killing vector fields on S^3 (see [3], [5]), and each integral curve of xi (or xj or xk) is a geodesic in S^3 and a circle in $\mathbf{H}$. We shall prove the followings:

THEOREM A. *Let* (M^3, f) *be an oriented hypersurface in the quaternions* $\mathbf{H}$ *and* ξ *a global normal vector field of* M^3 *in* $\mathbf{H}$. *If each* 1*-form of the associated dual frame field on* M^3 *is a contact form on* M^3 *and each integral curve of the associated orthonormal frame field is a circle in* $\mathbf{H}$, *then*

(1) M^3 *is locally isometric to a* 3*-dimensional round sphere in* $\mathbf{H}$ *and the immersion* f *is totally umbilic,*

or

(2) M^3 *is locally isometric to* $S^1\times\mathbf{R}^2$ (S^1 *is a circle) and the immertion* f *is a locally product one.*

TEEOREM B. *Let* (M^3, f) *be an oriented complete hypersurface in the quaternions* $\mathbf{H}$ *and* ξ *a global unit normal vector field of* M^3 *in* $\mathbf{H}$. *If each integral*

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