TSUKUBA J. MATH.
Vol. 13 No. 1 (1989). 113–129

ON LORENTZ MANIFOLDS WITH ABUNDANT ISOMETRIES

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0. Introduction.

Let M be an n-dimensional Lorentz manifold with metric $\langle\ ,\ \rangle$ of signature $(-, +, \cdots, +)$. Then there is no r-dimensional isometry group whose isotropy subgroup at every point is compact for $n(n-1)/2+1<r\leqq n(n+1)2$ (c.f., [5], Proposition). In [6], we determined n-dimensional Lorentz manifolds M which admit an $n(n-1)/2+1$-dimensional isometry group with compact isotropy subgroup at every point for $n\geqq 4$.

The first purpose of this note is to determine simply connected M admitting an $n(n-1)/2$-dimensional isometry group with compact isotropy subgroup at every point for $n\geqq 4$ (see §2). We will prove the following Theorem A.

THEOREM A. *Let $(M, \langle\ ,\ \rangle)$ be a simply connected n-dimensional Lorentz manifold admitting a connected $n(n-1)/2$-dimensional isometry group with compact isotropy subgroup at every point in M $(n\geqq 4)$. Then M is isometric to the warped product manifold $(I\times N, -dt^2+\phi(t)ds_N^2)$ where I is an open interval and N is the simply connected $(n-1)$-dimensional Riemannian manifold with metric ds_N^2 of constant curvature and $\phi(t)$ is a positive function on I.*

For isometry groups whose dimension are less than $n(n-1)/2$, we will have the following proposition in §1.

PROPOSITION 1.1. *If $n\geqq 6$, there is no r-dimensional isometry group with compact isotropy subgroup at every point for $(n-1)(n-2)/2+3\leqq r\leqq n(n-1)/2-1$.*

In view of Proposition 1.1, it is natural to ask which Lorentz manifold of dimension n admits an $(n-1)(n-2)/2+2$-dimensional isometry group with compact isotropy subgroup. The second purpose of this note is to determine simply connected manifold M admitting an isometry group of dimension $(n-1)(n-2)/2+2$ with compact isotropy subgroup at every point (see §3). We will prove the following Theorem B.

Received August 24, 1987. Revised April 7, 1988.