TSUKUBA J. MATH.
Vol. 13 No. 1 (1989). 7—11

FACTORIZATION THEOREM FOR PERFECT MAPS BETWEEN METRIZABLE SPACES

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1. Introduction. We assume that all spaces are normal and all maps are continuous. We write $A\in\mathrm{ANR}$ for a space A if A is an ANR for the class of all compact metrizable spaces.

Given spaces X and A we write $\dim X\leqq A$ if for any closed subset F of X any map $f: F\rightarrow A$ can be extended to X. For a map $\xi: X\rightarrow X_0$ we write $\dim\xi\leqq A$ if $\dim\xi^{-1}(x_0)\leqq A$ for any $x_0\in X_0$. It is kown that a space X satisfies the relation $\dim X\leqq S^n$ for the n-sphere S^n if and only if X satisfies the inequality $\dim X\leqq n$ in the sense of the covering dimension.

Our purpose in this paper is to prove the following theorem;

THEOREM. *Let $A\in\mathrm{ANR}$, let ξ be a closed map of a space X into a paracompact space X_0, ζ be a perfect map of a metrizable space Z into a metrizable space Z_0, and let $f: X\rightarrow Z$ and $f_0: X_0\rightarrow Z_0$ be maps such that $\zeta f=f_0\xi$ and $\dim\xi\leqq A$. Then there are metrizable spaces Y and Y_0, a perfect map $\eta: Y\rightarrow Y_0$, and maps $g: X\rightarrow Y$, $g_0: X_0\rightarrow Y_0$, $h: Y\rightarrow Z$ and $h_0: Y_0\rightarrow Z_0$ such that $\eta g=g_0\xi$, $\zeta h=h_0\eta$, $hg=f$, $h_0g_0=f_0$, $\dim\eta\leqq A$, $w(Y_0)\leqq\max(w(X_0), w(Z_0))$, and $\dim Y_0\leqq\dim X_0$.*

$$\begin{tikzcd} X \arrow[r, "g"] \arrow[d, "\xi"'] & Y \arrow[r, "h"] \arrow[d, "\eta"] & Z \arrow[d, "\zeta"] \\ X_0 \arrow[r, "g_0"'] & Y_0 \arrow[r, "h_0"'] & Z_0 \end{tikzcd}$$

For a map $\zeta: Z\rightarrow Z_0$ we write $w(\zeta)\leqq\tau$ if there is an embedding $i: Z\rightarrow Z_0\times I^{\tau}$ such that $\zeta=\mathrm{pr}\, i$, where I^{τ} is the Tikhonov cube of weight τ and $\mathrm{pr}: Z_0\times I^{\tau}\rightarrow Z_0$ is the projection.

In [9] Pasynkov proved a similar theorem to the above theorem, in which he added the property that $w(\eta)\leqq\tau$, if $w(\xi)\leqq\tau$, in the case that X, X_0, Z, Z_0 are compact (which are not assumed to be metrizable).

However, in [7] Pasynkov stated that, if f is a perfect map between

Received October 22, 1987.