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APPROXIMATIVE SHAPE II —GENERALIZED ANRs—

By

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§ 0. Introduction.

This paper is a continuation of [38], in which we introduced approximative shape. In this paper we introduce many approximative shape properties for spaces. These are approximative shape invariants and unify generalized absolute neighborhood retracts.

In 1931 Borsuk introduced the notions of an absolute neighborhood retract and an absolute retract, in notations ANR and AR, for metric spaces, respectively. There are many generalizations of ANRs and ARs. In 1953 Noguchi [26], introduced the notions of an ε -ANR and an ε -AR for compact metric spaces. Gmurczyk [11, 12] studied some shape properties of ε -ANRs and ε -ARs. She introduced the terms of an approximative absolute neighborhood retract in the sense of Noguchi and an approximative absolute retract, in notations AANR_N and AAR, respectively, to replace Noguchi's less convenient names ε -ANR and ε -AR. Clapp [8] introduced an approximative absolute neighborhood retract in the sense of Clapp, in notation AANR_c, for compact metric spaces. Bogatyi [2] studied many properties of AANR_N, AANR_c and AAR. Kalini [14] introduced these notions for compact spaces, and Powers [28] for metric spaces. Mardešić [22] introduced the notion of approximative polyhedra. Recently Gauthier [9, 10] introduced AANE_N, AANE_c and AAE which are generalizations of an absolute neighborhood extensor and an absolute extensor for metric spaces.

In 1986 Borsuk introduced shape theory, which was then developed by many mathematicians. Shape theory gives us a method to investigate bad spaces and bad maps by means of the good homotopy category of polyhedra. We have many important notions in shape theory; for examples, movability (see [5], [20]), uniform movability (see [25]), strong movability (see [6], [24]), absolute neighborhood shape retracts (see [4], [23]) and so on (see [19]). These notions play fundamental roles in shape theory.

In [38] we introduced approximative shape. It gives us a method to investigate bad spaces and bad maps by means of the good category of polyhedra. In Received February 4, 1986.