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ON ARTINIAN QF-3, 1-GORENSTEIN RINGS

By

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(Dedicated to Professor Hisao TOMINAGA on his 60th birthday)

A noetherian ring is called 1-Gorenstein if it has the self-injective dimension at most one on both sides. A well known example of artinian QF-3, 1-Gorenstein rings is the triangular matrix ring over a QF ring, which is QF-2, that is, every indecomposable projective module has the simple socle. Conversely Sumioka [11] characterized such a ring as an artinian QF-3, 1-Gorenstein ring with QF maximal quotient ring. But an artinian QF-3, 1-Gorenstein ring has not necessarily the QF maximal quotient ring (see §4). On the other hand, Sumioka's result is a generalization of Harada's characterization of artinian QF-3 hereditary rings, which states that a connected artinian ring is QF-3 hereditary if and only if it is Morita equivalent to a triangular matrix ring over a division ring (cf. [3]). Our results in the present paper are closely related to their results mentioned above.

First we shall deal with artinian QF-3 hereditary rings, which were investigated by Harada [3] and Iwanaga [4]. Our result is as follows.

THEOREM I. *Let Λ be a connected artinian ring which is not a QF ring. Then the following conditions for Λ are equivalent.*

(1) *Λ is a QF-3 hereditary ring.*

(2) *Λ is Morita equivalent to a triangular matrix ring over a division ring.*

(3) *Λ is a (left and right) serial 1-Gorenstein ring.*

(4) *Λ is a QF-3, 1-Gorenstein ring with a simple projective left module.*

(5) *Λ is a QF-3, 1-Gorenstein ring with a simple injective left module.*

Next we shall deal with the following problem:

(*) To investigate the length of the socle of an indecomposable projective module over an artinian QF-3, 1-Gorenstein ring.

It is well known that every indecomposable projective module over Λ is distributive in the sense of [1] if Λ is a representation-finite algebra over an algebraically closed field (cf. [6]). So it seems that it is worth studying artinian QF-3, 1-Gorenstein rings over which every indecomposable projective module is distributive. Our answer to the problem (*) is given by the following theorem.

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