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PARTIAL COXETER FUNCTORS AND STABLE EQUIVALENCES FOR SELF-INJECTIVE ALGEBRAS

By

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(Dedicated to Professor Hisao Tominaga on his 60th birthday)

Introduction.

The important notion of reflection functors was introduced into the representation theory of algebras by Bernstein-Gelfand-Ponomarev [8]. Those functors were defined only for hereditary tensor algebras given by quivers and species [12]. Then Auslander-Platzeck-Reiten [7] arranged the notion by non-diagramatic treatment so that it is possible to apply the concept for any algebras. Brenner-Butler [10] extended the Auslander-Platzeck-Reiten partial Coxeter functor and defined the tilting theory. Further, Happel-Ringel [15] generalized the Brenner-Butler tilting theory and studied tilted algebras.

We regard the tilting theory as a powerful method of deforming algebras and their module categories. A tilting functor, hower, is nothing but a Morita equivalence, for any self-injective algebra. Hence, it is natural to search for a way of applying the tilting theory to the study of self-injective algebras.

Let A be a basic indecomposable artin algebra. Denote by mod-A (resp. A-mod) the category of all finitely generated right (resp. left) A-modules. Let D: mod-$A\rightleftarrows A$-mod be the ordinary duality functor. In the following, we shall consider the trivial extension self-injective algebra $R=A\ltimes DA$ defined as follows: R is $A\oplus DA$ as an additive group and its multiplication is given by $(a,q)\cdot(a',q')=(a\cdot a',a\cdot q'+q\cdot a')$ for any $(a,q),(a',q')\in A\oplus DA=R$.

In the paper [19], Tachikawa started in the study of self-injective algebras R, and in [20], he has proved that $\underline{\text{mod}}$-R is equivalent to $\underline{\text{mod}}$-S ($S=B\ltimes DB$) if A is hereditary tensor algebra and B is given by reflection procedure from A. Here $\underline{\text{mod}}$-R is the projectively (=injectively) stable category of mod-R in the sense of Auslander.

Let $e\in A$ be a primitive idempotent such that eA is simple non-injective and $\tau^{-1}eA\otimes_A DA=0$, where τ^{-1} (resp. τ) denotes the Auslander-Reiten translation TrD (resp. DTr). By putting $T_A=(1-e)A\oplus\tau^{-1}eA$ and $B=\text{End}(T_A)$, the Auslander-Platzeck-Reiten partial Coxeter functor is defined to be the functor $\text{Hom}_A(T,?)$:

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