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EQUIVALENCE PROBLEM AND AUTOMORPHISM GROUPS OF CERTAIN COMPACT RIEMANN SURFACES

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Introduction.

Let V be the compact Riemann surface defined by the equation:

$$y^{n}=f(x),$$

where n is a positive integer and f is a rational function of x. For such V, there is a conjecture:

CONJECTURE: *The moduli of such V can be determined by the branch locus of the map $(x, y)\in V\rightarrow x\in \mathbb{P}^{1}$.*

Here, $\mathbb{P}^{1}$ is the complex projective line. The purpose of this paper is to give affirmative answers to the conjecture under various conditions. It is separated into 3 parts.

In Part 1, we assume that $n=p$ is a prime number and obtain a result. Recently, Kato [2] has improved this result extensively.

In Part 2, we assume that $f(x)$ is a polynomial of degree p with p a prime number, and obtain a result.

In Part 3, we assume:

$$f(x)=(x-\alpha_{1})\cdots(x-\alpha_{n}),$$

where $\alpha_{1}, \cdots, \alpha_{n}$ are mutually distinct complex numbers, and obtain an affirmative answer to the conjecture. In this case, the result can be extended to the case of non-singular hypersurfaces in $\mathbb{P}^{r+1}$, the $(r+1)$-dimensional complex projective space.

Corresponding to each case, we naturally obtain information on the automorphism group $\mathrm{Aut}(V)$ of V.

We note such compact Riemann surfaces were treated by Picard [8], Lefschetz [4], Shimura [9] and Kuribayashi [3] in connection with the study of Jacobian varieties and the concrete construction of some modular fuctions of several variables.

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