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ON PROJECTIVE NORMALITY AND DEFINING EQUATIONS OF A PROJECTIVE CURVE OF GENUS THREE EMBEDDED BY A COMPLETE LINEAR SYSTEM

By

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Introduction. Let $\phi_L : C \hookrightarrow \boldsymbol{P}^{h^0(L)-1}$ be the projective embedding of a complete non-singular curve C of genus g by means of $\Gamma(L)$, where L is a very ample invertible sheaf on C. We will study the homogeneous coordinate ring and the ideal of definition $I(L)$ of $\phi_L(C)$ in the case $g=3$. Our results are summarized in the following table. (If the genus of C is less than three, answers to the same kind of problems are easy.) In the table we will say that the homogeneous ideal $I(L)$ is generated strictly by its elements of degrees $\nu_1, \cdots, \nu_m$ if $I(L)$ is generated by its elements of degrees $\nu_1, \cdots, \nu_m$ and $I(L)$ is not generated by its elements of degrees $\nu_1, \cdots, \hat{\nu}_j, \cdots, \nu_m$ for any ν_j $(1 \leqq j \leqq m)$, where $\hat{\nu}_j$ means that ν_j is omitted.

$d \leqq 3$	There is no very ample invertible sheaf of degree $d \leqq 3$ on C.
$d=4$	If C is hyperelliptic, then C has no very ample invertible sheaf of degree 4. If C is non-hyperelliptic, then there is only one very ample invertible sheaf of degree 4 on C, which is the canonical sheaf ω_C. $\phi_{\omega_C}(C)$ is projectively normal. The homogeneous ideal $I(\omega_C)$ is generated strictly by its element of degree 4.
$d=5$	There is no very ample invertible sheaf of degree 5 on C.
$d=6$	The set of very ample invertible sheaves of degree 6 on C coincides with $$\mathrm{Pic}^6(C) - \{\omega_C(P+Q) \mid P, Q \in C\}\ .$$ If C is hyperelliptic, then for a very ample invertible sheaf L of degree 6 on C, $\phi_L(C)$ is not projectively normal and the homogeneous ideal $I(L)$ generated strictly by its elements of degrees 2 and 4. If C is non-hyperelliptic, then for a very ample invertible sheaf L of degree 6 on C, $\phi_L(C)$ is projectively normal and the homogeneous ideal $I(L)$ is generated strictly by its elements of degree 3.

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