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INVARIANTS OF FINITE GROUPS GENERATED BY PSEUDO-REFLECTIONS IN POSITIVE CHARACTERISTIC

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Introduction

Let R be a commutative ring, and let V be a finitely generated free R-module. Let $R[V]$ be a polynomial ring over R associated with V. Then a finite subgroup G of $GL(V)$ acts naturally on $R[V]$. We denote by $R[V]^G$ the ring of invariants of $R[V]$ under the action of G.

Let $R=k$ be a field and suppose that $|G|$ is a unit of k. It is known ([**4**], [**9**], [**3**], [8]) that $k[V]^G$ is a polynomial ring if and only if G is generated by pseudo-reflections in $GL(V)$.

But, in the case where $|G|\equiv 0$ *mod char*(k), there are only the following results:

(1) L. E. Dickson [**5**]; $\boldsymbol{F}_q[T_1, \cdots, T_n]^{GL(n,q)}$ and $\boldsymbol{F}_q[T_1, \cdots, T_n]^{SL(n,q)}$ are polynomial rings, where $\boldsymbol{F}_q$ is the finite field of q elements.

(2) M.-J. Bertin [**1**]; $\boldsymbol{F}_q[T_1, \cdots, T_n]^{Unip(n,q)}$ is a polynomial ring, where

$$Unip(n, q)=\left\{\sigma\in GL(n, q):\ \sigma=\begin{bmatrix} 1 & & 0 \\ & \ddots & \\ * & & 1 \end{bmatrix}\right\}.$$

(3) J.-P. Serre [8]; (i) If $k[V]^G$ is a polynomial ring, then G is generated by pseudo-reflections in $GL(V)$. (ii) $\boldsymbol{F}_q[T_1, T_2, T_3, T_4]^{O_4^+(\boldsymbol{F}_q)}$ is not a polynomial ring, where $O_4^+(\boldsymbol{F}_q)$ is the orthogonal group and $char(\boldsymbol{F}_q)\neq 2$.

The purpose of this paper is to determine finite irreducible subgroups G of $GL(V)$ such that $k[V]^G$ are polynomial rings in the case where $|G|\equiv 0$ *mod char*(k). Let V be an n-dimensional vector space over a finite field k of characteristic p and let G be a subgroup of $GL(V)$. Then our results are the following

[I] *If G is a transitive imprimitive group generated by pseudo-reflections, then $k[V]^G$ is a polynomial ring.*

[II] *Suppose that $p\neq 2$, $n\geqq 3$ and G is an irreducible group generated by transvections. Then $k[V]^G$ is a polynomial ring if and only if G is conjugate in $GL(V)$*

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