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A CORRESPONDENCE BETWEEN OBSERVABLE HOPE IDEALS AND LEFT COIDEAL SUBALGEBRAS

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1. Introduction. Let H be a commutative Hopf algebra over a field k with the antipode S. A Hopf ideal I of H is observable if it satisfies the following condition; for any finite dimensional right (resp. left) H/I—comodule V, there exists a right (resp. left) H-comodule W with the the structure map λ_W (resp. ρ_W) and an injective H/I—comodule map θ: $V\rightarrow W$, viewing W as a right (resp. left) H/I—comodule via

$$W \xrightarrow{\lambda_W} W\otimes H \xrightarrow{1\otimes\pi} W\otimes H/I$$

$$(\text{resp. } W \xrightarrow{\rho_W} H\otimes W \xrightarrow{\pi\otimes 1} H/I\otimes W),$$

where, in the following too, π: $H\longrightarrow H/I$ is a canonical Hopf algebra map and $\otimes$ means a tensor product over k. If G is an affine algebraic group defined over k and K is its closed subgroup defined over k, then $I=I(K)$, the ideal of the definition for K, is observable in $H=k[G]$, the coordinate ring of G over k, if and only if K is an observable subgroup of G in sense of [1].

A subalgebra of H which is also a left coideal of H is called a left coideal subalgebra.

In this paper, we give a bijective correspondence between observable Hopf ideals and left coideal subalgebras A which satisfies

$$(*) \qquad A=\mathrm{Ker}\,(H \underset{in_2}{\overset{in_1}{\rightrightarrows}} H\otimes_A H)$$

and a canonical construction of W from V. In the last section where we assume that a ground field k is algebraically closed and H is a finitely generated domain over k as an algebra (which we call an affine Hopf domain), we show that the condition (*) on A is equivalent to the fact that if $a\in A$ is a unit in H, then a is a unit in A. Moreover, in this case, A is finitely generated over k as an algebra (which we call an affine k-algebra as usual.) [2] shows that A is affine if and only

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