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ON THE NULLITIES OF KÄHLER C-SPACES IN $P_N(C)$

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Let M be a Kähler C-space which is holomorphically and isometrically imbedded in an N-dimensional complex projective space $P_N(\boldsymbol{C})$. Then M is a minimal submanifold of $P_N(\boldsymbol{C})$. Let $n_a(M)$ be the analytic nullity of M which was defined in [2]. We know that the nullity $n(M)$ of M is equal to $n_a(M)$ if M is a Hermitian symmetric space (Kimura [2]). In this note we prove that $n(M)=\mathrm{n}_a(M)$ for any Kähler C-space M.

By a theorem of Simons [5], the nullity of a Kähler submanifold coincides with the real dimension of the space of holomorphic sections of a normal bundle of the submanifold. Put $M=G/U$ where G is a complex semi-simple Lie group and U is a parabolic subgroup of G. By a result of Nakagawa and Takagi [4], we know that every imbedding of M in $P_N(\boldsymbol{C})$ is induced by a holomorphic linear representation of G. From this result we see that the normal bundle $N(M)$ over M is a homogeneous vector bundle.

We prove Theorem 1 which generalizes the generalized Borel-Weil theorem of Bott [1]. Applying the theorem to calculate the dimension of the space of holomorphic sections of $N(M)$ and prove that $n(M)=n_a(M)$.

The auther proved the above result before Proffesor Takeuchi gave another proof of it. His proof does not use Theorem 1 and is more simple than our proof (c.f. Takeuchi [6]).

§1. The generalization of Bott's result.

Let G be a simply connected compact semi-simple Lie group with Lie algebra $\mathfrak{g}$. Take a Cartan subalgebra $\mathfrak{h}$ of $\mathfrak{g}$. Denoto by Δ the root system of $\mathfrak{g}$ with respect to $\mathfrak{h}$. We fix a linear order on the real vector space spaned by the elements $\alpha\in\Delta$. Let Δ^+ (resp. Δ^-) be the set of all positive (resp. negative) roots. Let $\Pi=\{\alpha_1,\cdots,\alpha_l\}$ be the fundamental root system, where l is the rank of $\mathfrak{g}$ and Π_1 be a subsystem of Π. We put

$$\Delta_1=\{\alpha\in\Delta\,;\ \alpha=\sum_{i=1}^{l} m_i\alpha_i,\ m_j=0 \quad \text{for any} \quad \alpha_j\notin\Pi_1\}$$

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