A generalization of P. Roquette's theorems

Dedicated to Professor Yoshie Katsurada on her 60th birthday

By Tadashige OKADA and Ryô SAITÔ

Introduction

Throughout this paper, we assume that every ring has an identity 1, every module over a ring is unitary and a ring extension A/B has the same identity 1. For a commutative ring R, we consider only R-algebras which are finitely generated as R-modules. By [5], an R-algebra Λ is called left semisimple if any finitely generated left Λ-module is (Λ, R)-projective. Similarly we can define right semisimple R-algebras, and an R-algebra Λ is called semisimple if Λ is left and right semisimple. When R is indecomposable, an R-algebra Λ is called simple if (1) Λ is semisimple, (2) there exists left Λ-module ${}_{\Lambda}E$ which is finitely generated projective completely faithful and (Λ, R)-irreducible ([12]). We call an R-algebra Δ a division R-algebra if Δ is semisimple and (Δ, R)-irreducible. Obviously division algebras are simple algebras.

The followings are well known. Let K be a field (a field means commutative field) and let A be a finite dimensional central simple K-algebra. Then there exists a central division K-algebra D such that $A \cong (D)_n$ ($n \times n$ full matrix ring over D), and the free rank of D over $K([D:K])$ equals s^2 where $s(\geqq 1)$ is an integer. This s is called the Schur index of A and D is called a division algebra to which A belongs.

Let Δ be a division R-algebra and Λ be a simple R-algebra. If there exists a Morita module ${}_{\Lambda}M_{\Delta}$ ([9]), Δ is called a division R-algebra to which Λ belongs. By [12], any simple R-algebra belongs to some division R-algebra. Now, let R be a Hensel ring ([2], [10]) and Λ be a simple R-algebra. Then $\Lambda \cong (\Delta)_n$ where Δ is a division R-algebra to which Λ belongs. Moreover, Δ is uniquely determined up to isomorphisms and n is uniquely determiend ([12]).

The purpose of this paper is to extend some properties with respect to the Schur index concerning fields to the case of that R is a Noetherian Hensel ring.

We prove the followings.

THEOREM 2.2. *Let R be a semilocal ring (not necessarily Noetherian*