BASE CONNECTIONS IN A SPECIAL KAWAGUCHI SPACE

By

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Introduction. In a FINSLER space the arc-length of a curve, $x^i = x^i(t)$, is given by the integral

$$\int F(x^1, x^2, \ldots\ldots, x^n ; x'^1, x'^2, \ldots\ldots, x'^n)dt \, .$$

The function F is positively homogeneous of degree one with respect to $x'^1, x'^2, \ldots\ldots, x'^n$.

Prof. A. KAWAGUCHI(1) first investigated a space involving metric tensors whose components are functions of not only x^i and x'^i but of higher derivatives $x'', \ldots\ldots, x^{(r)}$ as well. Accordingly we give the name a special KAWAGUCHI space to the manifold associated with the integral $\int F(x, x', x'', \ldots\ldots, x^{(m)})\, dt$. He has developed the base connections of order r and of dimension n in his space $K_n^{(r)}$:

$$\overset{s+1}{p}{}^{\nu} = \frac{\overset{(s)(s)}{\delta\, p^{\nu}}}{dt} = \frac{d\overset{s}{p}{}^{\nu}}{dt} + \overset{(s)}{\Gamma}{}^{\nu}_{\lambda\mu}\overset{s}{p}{}^{\lambda}\overset{1}{p}{}^{\mu} - s\overset{(s)}{\Gamma}_{\mu}\overset{s}{p}{}^{\nu}\overset{1}{p}{}^{\mu} \; (2) \, ,$$

$$s = 1, 2, \ldots\ldots, \quad r-1 \, .$$

H. V. CRAIG(3) and J. L. SYNGE(4) have found various intrinsic vectors and covariant differentials in a special KAWAGUCHI space, where the

(1) A. KAWAGUCHI, [I] Die Differentialgeometrie in der verallgemeinerten Mannigfaltigkeit, Rendiconti del Circolo Mat. di Palermo, vol. LVI (1932), pp. 245–276.

(2) See [I], p. 255.

(3) H. V. CRAIG, [II] On a generalized tangent vector, American Journal of Math., vol. LVII (1935), pp. 456–462; [III] On the solution of the EULER equation for their highest derivatives, Bulletin of the Amer. Math. Soc., vol. XXXVI (1930), pp. 558–562.

(4) J. L. SYNGE, [IV] Some intrinsic and derived vectors in a KAWAGUCHI space, American Journal of Math., vol. LVII (1935), pp. 677–691.