ON THE MAXIMAL SPECTRALITY

By

Takasi ITŌ

Let R be a Hilbert space and $\mathfrak{B}$ a totally additive set class in an abstract space Ω. Asystem of proiection operators $E(\Phi)\,(\Phi\in\mathfrak{B})$ in R is called a *spectrality*[1] in R on $\mathfrak{B}$ if (1) $E(\Phi)+E(\Phi^C)=1$, and (2) $E\left(\sum_{i=1}^{\infty}\Phi_i\right) = \bigcup_{i=1}^{\infty} E(\Phi_i)$. We say a spectrality $E(\Phi)\,(\Phi\in\mathfrak{B})$ is *maximal* (due to Prof. NAKANO's suggestion) if

1) for any finite measure ν on $\mathfrak{B}$ we can find an element $x\in R$ such that $\nu(\Phi)=\|E(\Phi)x\|^2\,(\Phi\in\mathfrak{B})$, and

2) $\mathfrak{R}_E$ is a simple ring, where $\mathfrak{R}_E$ is a closed projection operator ring[2] generated by $\{E(\Phi);\ \Phi\in\mathfrak{B}\}$.

$\mathfrak{R}_E$ is *simple*[3] if and only if for any projection operator P that is commutative to $\mathfrak{R}_E$ we have $P\in\mathfrak{R}_E$.

In this paper we shall show that for any given Ω and $\mathfrak{B}$ we can construct a Hilbert space R and a maximal spectrality $E(\Phi)\,(\Phi\in\mathfrak{B})$ in R on $\mathfrak{B}$, and moreover R and $E(\Phi)\ (\Phi\in\mathfrak{B})$ are determined uniquely within an unitary isomorphism (Theorem 1). Conversely for any given R we can find Ω and $\mathfrak{B}$ for which there exists a *discrete* maximal spectrality in R on $\mathfrak{B}$. But it is known in WECKEN [1] that if the dimension of R is cotinuum, there exists in R a maximal spectrality on the Borel sets in the real line. If R is separable, we can prove that there is no maximal spectrality other than a discrete one (Theorem 2).

Theorem 1. *For any given Ω and $\mathfrak{B}$ we can construct a Hilbert space R and a maximal spectrality $E(\Phi)\,(\Phi\in\mathfrak{B})$ in R. Furthermore such R and $E(\Phi)\,(\Phi\in\mathfrak{B})$ are determined uniquely within unitary isomorphism.*

The method of the proof is essentialy same as in [1], so we give an outline only, about details refer [1] or NAKANO [2] Chap. V.

Let $\mathfrak{M}_{\mathfrak{B}}$ be the totality of finite measures on $\mathfrak{B}$. From the property of $\mathfrak{M}_{\mathfrak{B}}$ as a Boolian lattice and Maximal theorem we can find a maximal

1) cf. [2] §28.
2) cf. [2] §14.
3) cf. [2] §20.