Some congruence theorems for closed hypersurfaces in Riemann spaces

(The continuation of Part III)

Dedicated to the memory of Professor Dr. Heinz Hopf

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Introduction. This is the continuation of the previous paper ([1])[1] given by H. Hopf and the present author. In [1], considering an $(m+1)$-dimensional orientable Riemann space S^{m+1} with constant curvature of class C^ν $(\nu \geqq 3)$ which admits a one-parameter group G of isometric transformations, we proved the following

THEOREM. Let W^m and $\overline{W}^m$ be two orientable closed hypersurfaces in S^{m+1} which do not contain a piece of a hypersurface covered by the orbits of the transformations and p $\bar{p}$ be the corresponding points of these hypersurfaces along an orbit, and $H_r(p)$ and $\overline{H}_r(p)$, $r=1, \cdots, m$ be the r-th mean curvatures of these hypersurfaces at p and $\bar{p}$ respectively. Assume that in case $r \geqq 2$, the second fundamental form of $W^m(t) \overset{\text{def.}}{=} (1-t)W^m + t\overline{W}^m$, $0 \leqq t \leqq 1$, is positive definite. If the relation $H_r(p)=\overline{H}_r(p)$ holds for each point $p \in W^m$, then W^m and $\overline{W}^m$ are congruent mod G.

In the present paper, we shall cancel the assumption that the transformations are isometric, in fact, under a group G of essentially arbitrary transformations it is the purpose of the present paper to generalize the above theorem. Especially, in case of $r=m$, that is, the general theorem relating to the Gauss curvature was already proved in the previous paper [2].

§1. A certain integral form for two closed hypersurfaces. We suppose an $(m+1)$-dimensional orientable Riemann space S^{m+1} with constant curvature of class C^ν $(\nu \geqq 3)$ which admits an infinitesimal transformation

$$\bar{x}^i = x^i + \xi^i(x)\delta\tau \tag{1.1}$$

(where x^i are local coordinates in S^{m+1} and ξ^i are the components of a contravariant vector ξ). We assume that orbits of the transformations generated by ξ cover S^{m+1} simply and that ξ is everywhere continuous and $\neq 0$. Let us choose a coordinate system such that the orbits of transfor-

1) Numbers in brackets refer to the references at the end of the paper.