Remarks on modified symmetrizers for 2×2 hyperbolic mixed problems

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§ 1. Introduction and results

In this paper we are concerned with an existence theorem of a solution $u\in H_{1,-1;\gamma}(R^1\times\Omega)$ of the boundary value problem (P, B):

$$P(x, D)u(x) = f(x) \qquad \text{in } R^1\times\Omega,$$
$$B(x, D)u(x) = g(x) \qquad \text{on } \Gamma,$$

where $f\in H_{0,\gamma}(R^1\times\Omega)$ and $g\in H_{1/2,\gamma}(\Gamma)$. Here we assume that P is an x_0 -hyperbolic 2×2 system of pseudo-differential operators of order 1 and B is a 1×2 system of those of order 0 on the smooth boundary Γ of $R^1\times\Omega$.

While we try to extend the results in [7, section 7] to more general cases being inspired by the works of R. Agemi [2] and S. Miyatake [6], we find that there are certain gaps between L^2-well posedness for (P, B) (see [4]) and their conditions which is described in terms of symbols of P and B. In the present note, applying a concept of modified symmetrizers, we shall clarify the differences mentioned above and difficulties of mixed problems for hyperbolic systems. By localizations and coordinate transformations we may restrict ourselves to the case where

$$R^1\times\Omega = R^{n+1}_{+} = \left\{x=(x', x_n)\,;\; x_n>0\right\},$$
$$\Gamma = \left\{(x', 0)\,;\; x'=(x_0, x'')\in R^n\right\}.$$

Let (τ, σ, λ) be a covariable of $x=(x_0, x'', x_n)$ such that $\mathrm{Im}\,\tau\leq 0$. We assume that symbols of the principal part P^o of P and B are independent of x if $|x|$ is sufficiently large, homogeneous in (τ, σ, λ) and (τ, σ) respectively, analytic in τ and the determinant $\det P^o$ of P^o is an x_0-strictly hyperbolic polynomial of order 2. Moreover Γ is non-characteristic with respect to $\det P^o$ and $B(x', \tau, \sigma)$ is of rank 1 for any $(x', \tau, \sigma)\in R^n\times(C\times R^n\backslash 0)$. Finally any problems $(P, B)_x$ obtained by freezing their coefficients at $x\in\Gamma$ are L^2-well posed.

As it is well known, the difficulties in our problem (P, B) arise from the following: there is a point $(x^o, \tau^o, \sigma^o)\in\Gamma\times(R^n\backslash 0)$ such that the characteristic equation $\det P^o\,(x^o, \tau^o, \sigma^o, \lambda)=0$ has a real double root $\lambda=\lambda^o$ and