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Parametrices for pseudo-differential equations with double characteristics II.

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1. Introduction.

In our previous paper [11] we constructed parametrices for some double characteristic pseudo-differential operators, whose characteristic sets are closed conic manifolds of codimension 2 in the cotangent space. In this paper we shall show that the condition of Theorem 1.2 in [11] is necessary and sufficient condition for existence of parametrices.

In order to describe the result more precisely we must recall some notations and hypothese. Let X be a paracompact C^{∞} manifold of dimension n and $T^{*}(X)\backslash 0$ be the cotangential space minus the zero section. $P(x, D)$ is a properly supported classical pseudo-differential operator on X of order m. We denote the principal symbol of P by $p_{m}(x, \xi)\in C^{\infty}(T^{*}(X)\backslash 0)$. For arbitrary $C^{\infty}(T^{*}(X)\backslash 0)$ functions f and g we denote the Poisson bracket of f and g by $\{f, g\}$ and the Hamilton vector field of f by H_{f}. For a non-negative integer k and a connected closed conic non-involutory submanifold Σ of $T^{*}(X)\backslash 0$ with codim $\Sigma=2$, we use the notations $M^{m,k}(\Sigma, X)$ and $\sigma(P)$ for $P\in M^{m,k}(\Sigma, X)$ when k is odd. These notations are defined in definition 1.1 of [11].

We consider the following properly supported pseudo-differential operator $L(x, D)$ with double characteristics given by

$$L(x, D)=(P\cdot Q)(x, D)+R(x, D). \tag{1.1}$$

Here $P\in M^{m_{1},k}(\Sigma, X)$, $Q\in M^{m_{2},k}(\Sigma, X)$ and $R\in M^{m_{1}+m_{2}-1,k-1}(\Sigma, X)$.

In the following theorem we write $A\equiv B$ for operators A and B; $\mathscr{D}'(X)\rightarrow \mathscr{D}'(X)$ if $A-B$ is an integral operator with the C^{∞}-kernel. We also write diag $(V)=\{(\rho, \rho)$; $\rho\in V\}\subset(T^{*}(X)\backslash 0)\times(T^{*}(X)\backslash 0)$ for any conic subset V of $T^{*}(X)\backslash 0$. Our statement is the following

THEOREM. *Let $L(x, D)$ be a double characteristic pseudo-differential operator defined by* (1.1), *where k is an odd integer and $\sigma(P)=1$, $\sigma(Q)=-1$. We assume that $(H_{p_{m_{1}}})^{l}q_{m_{2}}(x, \xi)=0$ on Σ for $l=1, \cdots, k-1$ and $(H_{p_{m_{1}}})^{l} r_{m_{1}+m_{2}-1}(x, \xi)=0$ on Σ for $l=1, \cdots, k-2$ where $k>1$. Then the following statements are equivalent.*